Chapter 4 Changing speed –acceleration

Section 4.1 Motion with acceleration

Speed tells you how much position changes in time. And in our examples so far, we have assumed that speed is constant. But speed isn't always constant. If you have ever driven a car, you probably have some idea of what acceleration is; the pedal under the driver's right foot is commonly referred to by one of two names: the gas pedal or the accelerator. What happens to a car if the driver presses on the accelerator? It speeds up. The accelerator increases the speed. And that's exactly what acceleration is: a change in speed over a given time:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{t}} \qquad \text{Equation 4.1}$$

where $\boldsymbol{a}$ is the symbol for acceleration, $\boldsymbol{v}$ is the symbol for speed, and $\boldsymbol{t}$ is the symbol for time. This equation looks very similar to the one for velocity from Section 1.2, doesn't it? In fact, if you made a graph of speed vs time instead of position vs time, the slope of that graph would give the acceleration. Let's try it using Figure 4.1.

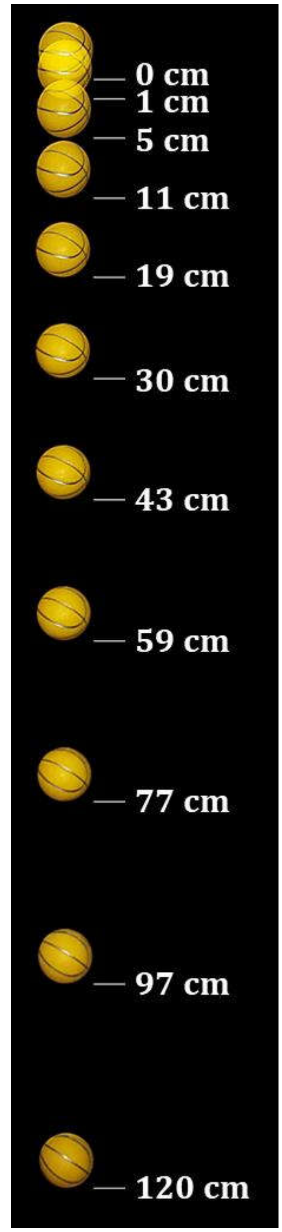
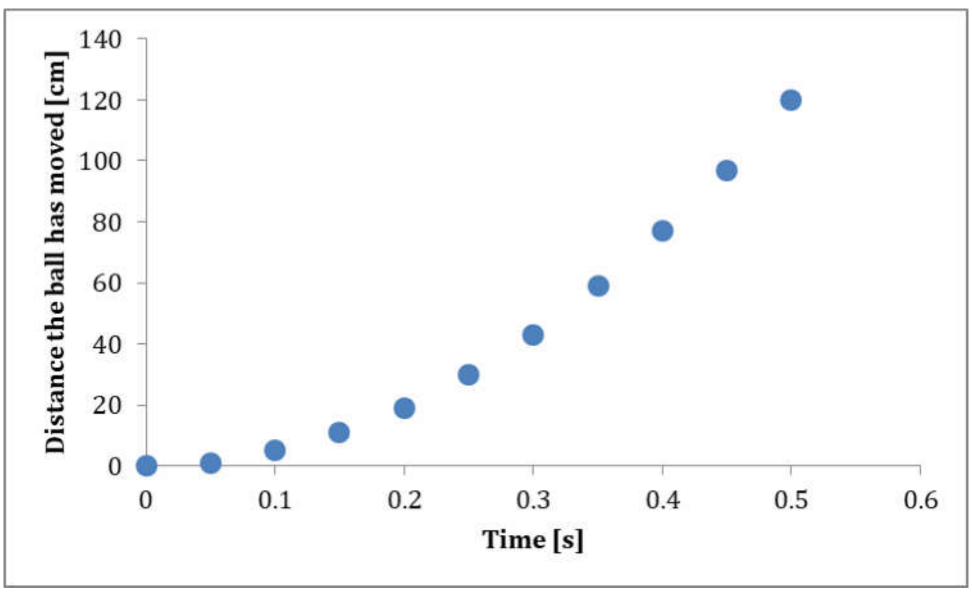
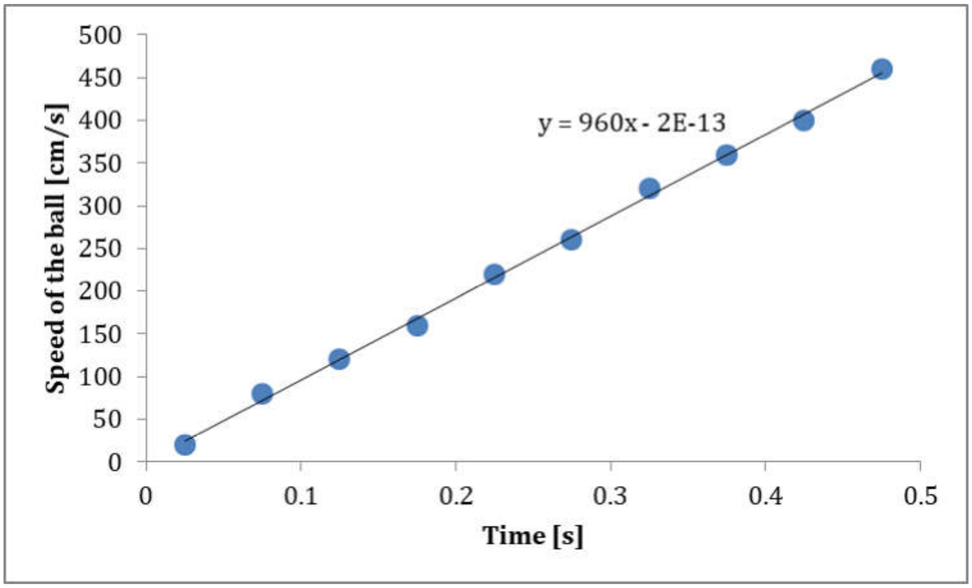


Figure 4.1 Images of a falling ball taken using a stroboscopic light source. Time between flashes is 0.05 s. (Credit: MichaelMaggs [CC BY-SA 3.0], from Wikimedia Commons, edited by Dean Stocker)

A position vs time graph for the ball would look like this:



We know that velocity is the slope of the line on this graph, but in this case it isn't really it's a line. It's a curve, which means that the slope is constantly changing. To get the velocity, we can use $\boldsymbol{v} = \frac{\Delta x}{\Delta t}$ for the space between each data point. So the speed between the first flash and the second is $\boldsymbol{v} = \frac{1-0}{0.05}$ cm/s, the speed between the second flash and the third is $\boldsymbol{v} = \frac{5-1}{0.05}$ cm/s, and so forth. If we do that, we find that the velocity vs time graph looks like this:



Note that the times used in the second graph are shifted compared to those in the first graph. The first speed data point is correct for the time halfway between the first two position data points. This graph does make a nice straight line, and according to the equation for the curve fit, the speed (y on the graph) is 960 times the time (x on the graph). The 2E-13 term (which is how engineers write 2×10^{-13}) gives information about the initial speed, which is very close to zero. The slope of this graph is $\frac{\Delta v}{\Delta t}$, which is acceleration. So the acceleration for the ball in Figure 4.1 is 960 cm/s^2. Converting to the standard unit of acceleration, we get 9.6 m/s^2, which is close to the 10 m/s^2 that we will use in this class for the acceleration of gravity at the earth's surface.

That looks like a crazy unit, m/s^2. After all, what is a second squared? It doesn't make sense to us. We have to think of it this way: it is a change in speed (measured for example in m/s), per time (in seconds). So it is measured in m/s per s, or (m/s)/s. That's two s's on the bottom, so m/s^2.

Let's look at it in an example we may be more familiar with. We often report acceleration of cars as the time necessary to attain a speed of 60 mph. For instance, Tesla boasts that its prototype accelerates to 60 mph in about 3 seconds. That's about 30 m/s in 3 seconds, so the speed increases 10 m/s every second: its acceleration from rest is about (10 m/s)/s, or 10 m/s^2. That means if you're in a Tesla accelerating from rest, your acceleration forward is the same as your acceleration downward would be if you were dropped from a cliff in that same Tesla!

Exercise 4.1 Let's look again at the rocket we saw in Section 1.2.

a. Do you think that the rocket has a large acceleration when it takes off?

b. After the rocket takes off is it still accelerating? Are there forces acting on it besides gravity?

c. After take-off, should its acceleration be more than gravitational acceleration, less, or the same?

d. In the video frame before the left most frame, the rocket was still at rest on top of the piece of plastic pipe. Please estimate the acceleration of the rocket as it takes off. Estimate doesn't mean "guess", it means "calculate it with the understanding that there are some uncertainties in your measurements and simplifications in your calculations." After you have estimated the rocket's acceleration at take-off, consider what would happen to you if you had that acceleration. You can look up how much acceleration a human can sustain.

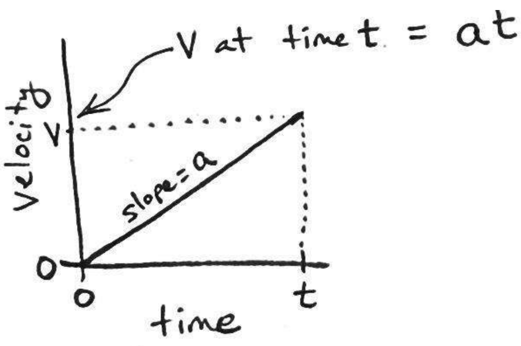
e. Estimate the speed between video frame 1 and video frame 2 more carefully than you did in chapter 1.4, by measuring the distance more accurately between rocket positions. Is the velocity constant? Is the rocket speeding up? Slowing down? How much time transpired between the speeds you calculated? Please estimate the acceleration of the rocket after taking off and compare it to gravitational acceleration. What do you find?

Section 4.2 **Motion – Distance with acceleration**

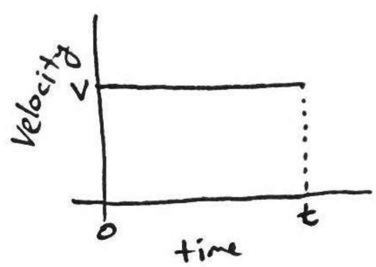
In Section 1.2 we saw an equation where position changes linearly with time. This is true when there is no acceleration, but as you can see from the curve in the graph of position vs time in Section 4.1, that's not true when an object is accelerating. In fact acceleration is the curvature of the position vs time graph. In Section 4.1 we started with measurements of distance and ended up with acceleration. Can we go backwards now, starting with acceleration and ending with distance? Here is the question:

An object starts at rest at position zero and accelerates at a constant rate a. What is the distance it moves in time t?

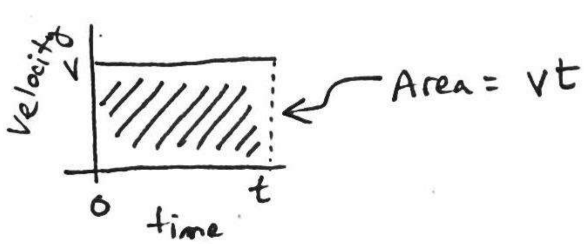
Graphs worked pretty well for us last time. Let's try that again. Acceleration is the slope on a graph of velocity vs time, so that graph would look something like this:



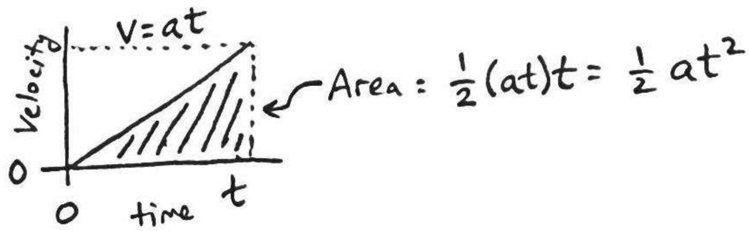
So that's what the velocity looks like in this situation. As time gets bigger, so does the velocity, in a straight line. But now how do we get the distance from that? If the velocity were constant, the graph would have looked like this:



And when velocity is constant, we know that the distance an object moves is ***v*Δ*t***. Look at the graph again. If ***v*** is up there at the flat line across the top, and **Δ*t*** is the time from 0 to the place marked "***t***" on the graph, then that "height" ***v*** multiplied by the "width" ***t*** is the area of the rectangle under the curve!



This holds true whether the area under the curve is a rectangle or a triangle or any crazy shape you can imagine. So if we could figure out the area under that triangular shape above, that would tell us the distance the object moved!



The area of a triangle is ½ of the base times the height, so in this case it would be (½)($\boldsymbol{t}$)($\boldsymbol{at}$), or more simply ½ $\boldsymbol{at^2}$. So for the object that started at rest, in time t it moves a distance ½ $\boldsymbol{at^2}$.

Remember, we assumed we were starting with no motion at t=0. It will be different if the initial velocity is not zero or if we start at a time not equal to zero.

Exercise 4.2 Assuming that there is no wind resistance, if I drop a rock from the top of a tall building…

a. How much time will it take to fall 1 meter? How fast will it be moving after it has fallen 1 meter?

b. How much time will it take to fall 10 meters? How fast will it be moving after it has fallen 10 meters?

c. Are your answers to part b ten times your answers to part a? Why or why not? Explain in words, not equations, as if you were explaining it to a 5-year-old!

d. Show that "½ at^2" has the units of distance.

Section 4.3 **Newton's Laws**

Back in Section 2.3 we learned Newton's Second Law, $F = ma$. Then in Section 3.5 we learned Newton's Third Law, that every force that acts on an object has to come from some other object, and that other object feels the same amount of force in the opposite direction. This is often described as "for every action there is an equal and opposite reaction." What happened to Newton's First Law? Actually, it's just one example of Newton's Second Law.

Acceleration is defined as a change in velocity over time. So if something is accelerating, that means that its velocity is changing. And if the velocity is NOT changing, that means the acceleration is zero. What is the force on an object that has zero acceleration?

Exercise 4.3 Look at Newton's Second Law again.

a. If you set the acceleration *a* equal to zero, what happens to the force *F*?

b. If the acceleration *a* is equal to zero, that means the velocity is not changing. In other words, it is constant. Use your answer to part a to write an "if-then" statement that says "If the velocity of an object is constant, the force on that object…."

What you have just written as the answer to Exercise 4.3, part b, is Newton's First Law! You worked it out yourself based only on Newton's Second Law. Newton's First Law is often stated like this: "An object at rest tends to stay at rest, and an object in motion tends to stay in motion, unless acted on by an external force." Those aren't the same words that you used in Exercise 4.3, but they mean the same thing.

Newton's Laws are closely related to what we have learned already about momentum. Momentum is conserved, which means that if there is no external force, the motion of an object will be constant. That's just Newton's First Law again! So there are many different (correct!) ways to view the same physical reality.

Let's try using Newton's Laws in problems. That means we will be using the "Force" lens, but you may find that you need to bring in other lenses to solve the problems as well.

Exercise 4.4 A baseball has a mass of 0.145 kg. A typical "fastball" pitch is approximately 90 mph, or 40 m/s. The time it takes the pitcher to throw the ball is only about 0.05 s.

a. What is the acceleration of the baseball as it is being pitched? You can ignore gravity for this part, and just look at the speed it has when it leaves the pitcher's hand after 0.05 s.

b. How much force does the pitcher apply to the baseball as he is throwing it?

c. A solid hit on a fastball can send it flying in the opposite direction at 50 m/s. How much change in velocity is that? (Remember to take direction into account!)

d. The ball is in contact with the bat for only approximately 0.0007 s. What is the average force of the bat on the ball?

Section 4.4 **Gravitational Potential Energy and Kinetic Energy**

We've been using the acceleration of gravity at the Earth's surface, 10 m/s^2, for a while now. Let's switch that up and look at gravitational force instead of gravitational acceleration. Go back through Chapter 2 if you need to, and find the relationship between force and acceleration.

Exercise 4.5 **What is the force of gravity at the Earth's surface on a cat with a mass of 4 kg?**

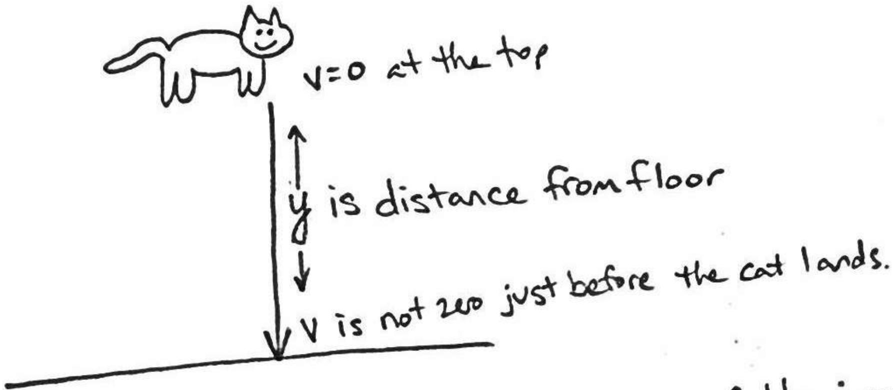
Exercise 4.6 **How much work is needed to lift a 4-kg cat 2 meters up into the air? Review Equation 2.4 if necessary.**

Consider your answer to Exercise 4.6. If the cat started at rest on the floor, and ended at rest 2 meters above the floor, and some amount of work was done on the cat, how was energy conserved? Conservation of energy says that the change in energy must be equal to the work put into the system. For any object of mass m, if you lift it upward some height h, the increase in gravitational potential energy of the object must be equal to the amount of work you did in lifting it:

$$\boldsymbol{E_g = mgy}$$ **Equation 4.2**

where $\boldsymbol{E_g}$ is gravitational potential energy, $\boldsymbol{m}$ is the mass of the object, $\boldsymbol{g}$ is the acceleration of gravity, and $\boldsymbol{y}$ is the height of an object...but the height above what? The ground? A table? The center of the earth? Actually it doesn't matter what you use as your "$\boldsymbol{y}$ = 0" reference point, as long as you consistently measure heights from the same point in any given problem. It usually simplifies the math if you choose the starting point or the ending point as $\boldsymbol{y}$=0.

What happens if the cat that you lifted 2 meters into the air in Exercise 4.6 decides that it doesn't want to be up there, and jumps back down? Gravitational potential energy transforms to kinetic energy from the time it begins the jump until the time just before it hits the ground. Assuming no outside forces are acting on the cat besides the force of gravity, the kinetic energy it has just before hitting the ground would be exactly equal to the gravitational energy it had just before it jumped. Let's see what that means about kinetic energy.



$h = \frac{1}{2}at^2$ can tell us the time of the jump.

↳ $t^2 = \frac{2y}{a} \longrightarrow t = \sqrt{\frac{2y}{a}}$

Also, we know that $v = at$, so $v = a\sqrt{\frac{2y}{a}} = \sqrt{2ay}$

I don't like square roots, so square both sides. $v^2 = 2ay$

And acceleration is g, so $v^2 = 2gy$

That tells us something about the velocity of the cat just before it touches the ground. But we are trying to learn about energy.

$$E_g(\textbf{initial}) = mgy = E_K(\textbf{final})$$

where E_K is kinetic energy. We just saw that for this problem $v^2 = 2gy$, and if we multiply both sides by m and shift it around a bit, we discover the relationship between mass, velocity, and kinetic energy:

$$mgy = \frac{1}{2}mv^2 = E_K(\textbf{final})$$

We used an example of a cat falling in a gravitational field, but actually this relationship between velocity, mass, and kinetic energy is true for any object in motion:

$$E_K = \frac{1}{2}mv^2 \qquad \textbf{Equation 4.3}$$

where E_K is kinetic energy, m is mass, and v is speed.

Exercise 4.7 **You have two rocks: rock A and Rock B. Rock A has twice the mass of rock B, and is also twice as high off the ground as rock B. What is the ratio of the energy of rock A to rock B?**

Exercise 4.8 **You have two rocks: Rock A has twice the mass of rock B, *and* is also moving at twice the speed of rock B. What is the ratio of the energies of rock A to rock B?**

Exercise 4.9 **You have two rocks dropped from the same height. Rock A has twice the mass of rock B. What is the ratio of their speeds when they hit the ground?**

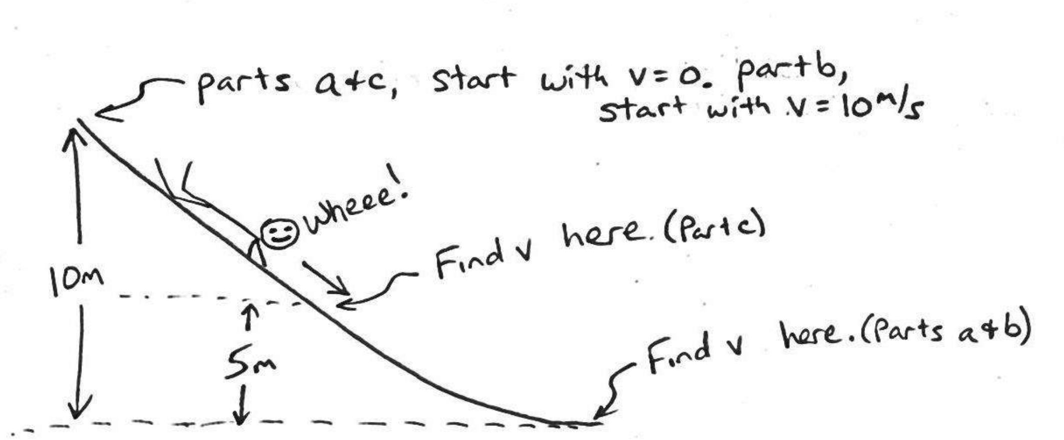
Exercise 4.10 **You have two rocks of equal mass. Rock A is dropped from twice the height as rock B. What is the ratio of their speeds when they hit the ground?**

So far in this section we have only looked at situations where the initial speed is zero. What if it isn't? Let's look at a problem where the initial speed is zero, and then change it so that the initial speed is not zero. Here is the question:

You go on a super slippery water slide that drops 10 meters to the ground.

a. ***What is your speed at the bottom of the slide if you start at rest at the top?***
b. ***What if you are a fast runner and were able to enter the top of the slide at an initial speed of 10 m/s? What would your speed be at the bottom?***
c. ***What is your speed half way down the slide if you start at rest at the top?***

Let's start with a picture.



We should try using the energy lens because we have gravitational potential energy at the beginning and kinetic energy at the end, and since the slide is "super slippery" that means we shouldn't lose any thermal heat along the way. So we conserve energy from the top to the bottom…

$$\text{Energy at the top} = \text{Energy at the bottom}$$

$$mgy(\text{at top}) + \frac{1}{2}mv^2(\text{at top}) = mgy(\text{at bottom}) + \frac{1}{2}mv^2(\text{at bottom})$$

That's a big equation! But we can simplify it, because ***v*** at the top is zero, at least for part a. And, we are free to choice any place on our drawing to make ***y*** = 0. It looks like the bottom of the ramp is a good choice. Also, all of the masses cancel out. So…

$$\cancel{m}gy(\text{at top}) + \frac{1}{2}\cancel{m}\cancel{v^2}^{\,0}(\text{at top}) = \cancel{m}g\cancel{y}^{\,0}(\text{at bottom}) + \frac{1}{2}\cancel{m}v^2(\text{at bottom})$$

$$gy(\text{at top}) = \frac{1}{2}v^2(\text{at bottom})$$

$$v = \sqrt{2gy} = \sqrt{2(10\,\text{m/s}^2)(10\,\text{m})} = 14\,\text{m/s}$$

14 m/s is approximately 30 mph, so a "medium car speed." That seems like a reasonable answer for part a.

Now, to part b…it's tempting to say that we can just add the starting speed of 10 m/s to our answer from part a to get the new answer, so 10 m/s + 14.1 m/s = 24.1 m/s. However, speed is not conserved; *energy* is conserved. To find the actual speed at the bottom, we need to use the same equation as before, but include the starting speed. So…

$$\cancel{m}gy(\text{at top}) + \frac{1}{2}\cancel{m}v^2(\text{at top}) = \cancel{m}g\cancel{y}^{\,0}(\text{at bottom}) + \frac{1}{2}\cancel{m}v^2(\text{at bottom})$$

$$gy(\text{at top}) + \frac{1}{2}v^2(\text{at top}) = \frac{1}{2}v^2(\text{at bottom})$$

$$v(\text{at bottom}) = \sqrt{2gy + v_{top}^2}$$

$$= \sqrt{2(10\,\text{m/s}^2)(10\,\text{m}) + (10\,\text{m/s})^2} = 17\,\text{m/s}$$

17 m/s is much slower than our initial thought that we should just add 10 m/s to our answer from part a! What was wrong with our initial reasoning about just adding the speeds? Do you remember that velocity increases with time if acceleration is constant? If you take a running start on the slide, your time on the slide is much less than if you started at rest. That means less time for acceleration! So a motion lens also shows that you can't just add the speeds together to get the right answer.

Exercise 4.11 Now try part of the slippery slide problem c on your own. Again, you should be able to use the same approach, but this time your final y value isn't at the bottom of the ramp. It's in the middle.

Exercise 4.12 **I throw a rock upward with an initial speed of 20 m/s.**

a. **How high does it get?**

b. **What is its speed when it is 10 m high?**

c. **What is its speed when it comes back down to the same height that it was when you threw it?**