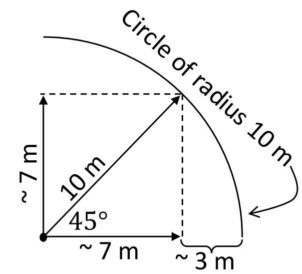
7.0 Components *We don't consider trigonometry until later. Please estimate visually for now.*

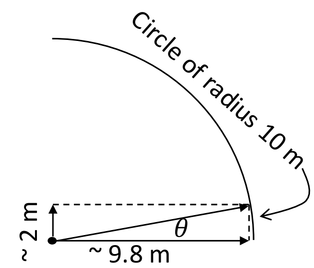
If we move 10 m horizontally in the $+\hat{x}$ direction, we don't change our vertical position, and if we move 10 m upward in the $+\hat{y}$ direction, our horizontal position doesn't change. If we were to move 10 m at a 45° angle above the horizontal, we might be tempted to think we move 5 m horizontally and 5 m vertically, but we can see that the $\hat{x}$ and $\hat{y}$ components of our displacement are clearly greater than 5 m, much closer to 7 m. We can express our displacement in $\hat{x}, \hat{y}$ component form:

$\overrightarrow{\Delta x} \sim 7m\,\hat{x} + 7m\,\hat{y}$



Similarly, if we moved in a direction that made a smaller angle, θ, with the horizontal, we could see the horizontal displacement is very close to the full 10 m:

$\overrightarrow{\Delta x} \sim 9.8\,m\,\hat{x} + 2\,m\,\hat{y}$



Exercise 1:

Your boat is capable of a speed of 4 m/s with respect to the water and you aim your boat northward across a river 100 m wide that is flowing 3 m/s west.

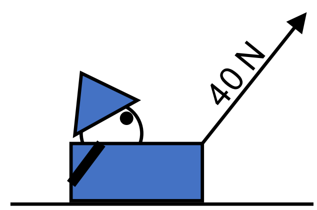
- Draw what someone sees from a bridge far overhead.
- What is the speed according to the person on the bridge?
- What is your total displacement in your trip across the river? Please put your answer in terms of north and west components.

It turns out that forces decompose into perpendicular components the same way. If you were to pull on a string with a 10 N force in the direction of the diagram above, at an angle, θ, with the horizontal. The force the string's tension applies to the point of contact is the same as

$\vec{F} \sim 9.8\,N\,\hat{x} + 2\,N\,\hat{y}$.

Exercise 2:

You are pulling a 10 kg sled on low friction ice by pulling an attached string with 40 N. Please make a good free body diagram for each of the following.



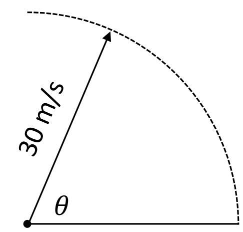
- If you pull directly upward, please find the resulting acceleration of the sled and $F_\perp$ provided by the underlying ice on the sled.
- If you pull in the $\hat{x}$ direction, please find the resulting acceleration of the sled and $F_\perp$ provided by the underlying ice on the sled.
- If you pull at an angle as indicated in the diagram, please find the resulting acceleration of the sled and $F_\perp$ provided by the underlying ice on the sled.
- If you pull at the smaller angle, θ as in the above diagram, please find the resulting acceleration of the sled and $F_\perp$ provided by the underlying ice on the sled.

How do we know to decompose the force in horizontal and vertical and not some other direction? Usually the direction of acceleration determines the way you break a force vector into perpendicular components. In the above example, the sled is accelerating in the $\hat{x}$ direction, so we know the $\hat{a} = 0$ in the $\hat{y}$ direction. Thus, $\hat{x}$-$\hat{y}$ is a good axis. We will see in the next section that if the surface is not horizontal, it is best to pick your directions according to the new direction of acceleration. You will see that when using the dynamics lens, *it's crucially important to identify the direction of acceleration in free body diagrams.*

Parabolic Trajectory

Throw a ball upward at an angle and watch its path from the side. You should notice that the ball traces a parabola. We can understand this by looking at the components of velocity. Once the ball leaves your hand, how many forces are acting on it, neglecting air friction? The only force, gravity, is downward so $a_x = 0$, and $a_y = -g$. Thus, we should decompose the velocity into $\hat{x}$ and $\hat{y}$ components, knowing that v_x is constant, and v_y decreases by $10\ \frac{m}{s}$ every second. As the ball progresses steadily in the $\hat{x}$ direction, the angle the velocity makes rotates downward, steadily changing the angle, consistent with a parabolic trajectory.

Exercise 3:



Let's say you could throw a rock at 30 m/s (about 66 mph!) at the angle indicated at right. We want to predict what will happen.

- Decompose the velocity appropriately.
- What will the velocity of the rock look like in 1s, 2s, 3 s?
- How long will it take for the rock to come to reach its highest point, and how long for it to come back down?
- How far will the rock move horizontally while it is in the air?
- Estimate the rock's maximum height.
- Draw the trajectory of the rock indicating the position at each second until it hits the ground.
- How would your answers be different if you threw the rock with the same initial velocity off the edge of a 100 m cliff?