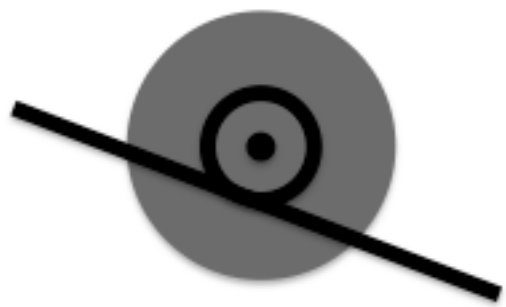
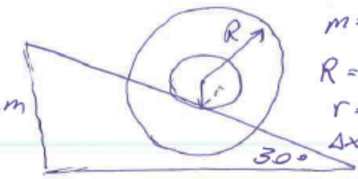
PS#10 Due Thursday, March 15 in class. Remember to start each question with a description of the lens and method.

1) A bicycle is a beautiful thing to me! This question is largely addressed through the "bicycle transmission" video in Week 10. Please see the video again if you are so inclined. However, I will address right here question H) What happens when you change the rear gear to twice the original radius? This is downshifting by a factor of two. At that moment, there is no immediate change of speed, so the rear wheel has the same rotational velocity. With twice the radius on the rear cog (gear), the chain must move twice the speed to keep up with the rotating wheel. Because the chain has the same tension on it (you are pushing with the same force on your feet), the power supplied by the chain ($P = F*v$) is doubled. Because the chain tension acts on the rear wheel at twice the radius, the chain's torque on the rear wheel doubles, doubling the force to the earth's surface (and the force of the earth's surface on the bike doubles). By doubling the force to the rear wheel, the bike will accelerate, and you've doubled the power delivered to the rear wheel. In order to move the chain twice as fast, you will need to spin your legs twice as fast, doubling your power output if you are able to continue pushing with the same force on your legs: $P = \tau\omega = Fv$. This is what we experience every day… if you are cruising at constant velocity on the freeway, your engine is not putting out very much power. But then you want to pass someone. You downshift to a lower gear (or your automatic transmission does that). The engine spins much faster, which you can hear. The power to the wheels increases greatly and you accelerate increasing the kinetic energy of your car. Same thing on a bicycle. Now can you answer the questions:

2) Remember the flywheel from a previous problem set?



a) $\Delta h = 2m$



$m = 3kg$
$R = .3m$
$r = .1m$
$\Delta x = 4m$

$\Delta PE = mg\Delta h = 3kg\, 10\, \frac{m}{s^2}\, 2m$
$= 60\, \frac{kg\, m^2}{s^2} = 60J$

b) Energy bus -

$I = \frac{1}{2}MR^2$
$v = \omega r$

$PE \Rightarrow KE_{linear} + KE_{rotation}$

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}\,\frac{1}{2}mR^2\omega^2$

$\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}\omega^2(r^2 + \frac{1}{2}R^2) = \frac{1}{2}\cancel{m}\omega^2 \underbrace{(10^{-2}m^2 + \frac{1}{2}.09m^2)}_{0.55\ m^2}$

$\omega^2 = \frac{2}{0.055\, m^2}\, 10\, \frac{m}{s^2}\, 2m \approx 727/s^2$

$\boxed{\omega_f \approx 27/s}$

$\omega_{ave} = \frac{\omega_f}{2} \approx 13.5/s$

$V_{ave} = \omega_{ave} \cdot r = 1.35\, m/s = \frac{\Delta x}{t}$

$t = \frac{\Delta x}{V_{ave}} = \frac{4m}{1.35\, m/s} \approx 3.0s$

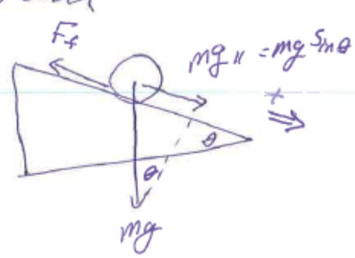
$\alpha = \frac{\Delta\omega}{\Delta t} \approx 9.1/s^2 \qquad a = \alpha r = 0.91\, m/s^2$

$\tau = \alpha I = \frac{9.1}{s^2}\,\frac{1}{2}mR^2 = \frac{9.1}{s^2}\left(\frac{1}{2}\right).3kg\,(.3m)^2 = 1.23\, Nm$

$F_f = ?\quad \tau = F_f r \quad F_f = \frac{\tau}{r} = 12.3N$ (up the hill)

This is now a dynamics problem

$\Sigma\vec{F} = m\vec{a} = mg_{\parallel} - F_f$



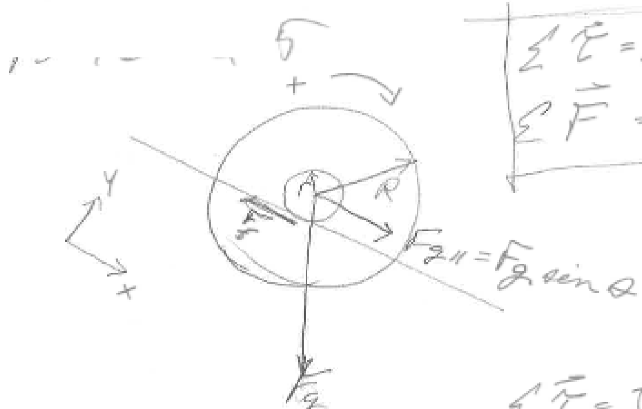
$= 15N - 12.3N \approx 2.7N$

$a = \frac{\Sigma\vec{F}}{m} = \frac{2.7N}{3kg} \approx 0.9\, m/s^2$

which is what we calculated previously.

Below is a different way to find the answers using a dynamics approach and solving the simultaneous equations. ***BUT at the very end, I show you how to solve it in just one line*** by just saying that at this instant in time, the wheel is pivoting ***around the point of contact*** and finding the torque = $F_{g(parallel)}$*r, and using the parallel axis theorem to find the moment of inertia of the wheel about this point.

$$\boxed{\begin{array}{l} \sum \vec{\tau} = I\vec{\alpha} \qquad a = \alpha r \\ \sum \vec{F} = m\vec{a} \end{array}}$$



$$\sum \vec{\tau} = I\vec{\alpha} \qquad \sum F = ma$$

$$F_f r = I\alpha \qquad F_{g\parallel} - F_f = ma$$

$$F_{g\parallel} - ma = F_f$$

$$F_{g\parallel} r - m\alpha r^2 = I\alpha \qquad F_{g\parallel} - m\alpha r = F_f$$

$$F_{g\parallel} r = I\alpha + mr^2\alpha$$

$$\boxed{\frac{F_{g\parallel} r}{\underbrace{I + mr^2}_{I_{PA}}} = \alpha}$$

$$\alpha = \frac{(\cancel{m}g \sin 30^\circ)(0.1\,m)}{\frac{1}{2}\cancel{m}(0.3\,m)^2 + \cancel{m}(0.1\,m)^2}$$ mass cancels

$$= \frac{5\,m/s^2\,(.1\,m)}{.055\,m^2}$$

$$= 9.1/s^2$$

$$a = \alpha r = 0.91\,m/s^2$$

$$\tau_{\text{Pt of contact}} = I_{\text{Parallel axis}}\,\alpha$$

$$F_{g\parallel} \cdot r = I_{PA}\,\alpha \qquad \alpha = \frac{F_{g\parallel} \cdot r}{I_{PA}}$$

3) The classic "notorious ladder problem" Please see the dedicated video.

4) In class I give you two ways to find a coefficient of friction between the masses and the spinning turntable: by measuring how far the disks move horizontally as they fall, and by measuring the inclination when the disks slip. Please get the measurements in class and do both calculations and see if you get reasonably close answers for the two different ways.

   1) I slowly raised one side of the turn table increasing the slope of the inclined plane of the surface. When the surface is inclined at an angle of about 35 degrees to the horizontal, the mass slid off the inclined surface. We recognize this as a statics problem where
      - The VERY important part of this problem is that we draw a good picture and ASK ourselves that important question. It's not in equilibrium, but accelerating down the incline at a 35 degree angle, so I have to pick my axis accordingly, and decompose gravity into the parallel and perpendicular directions.
      - the sum of the forces in the parallel direction yield: the force of friction = the parallel component of gravity, and
      - and the sum of the forces in the perpendicular direction yield: the normal force = the perpendicular component of gravity.
      - Hence, the normal force is ($\cos \theta$)mg, and the parallel component of gravity is ($\sin \theta$)mg.

      In solving the parallel force = ma = 0 (at the moment we break the static friction), we find that the coefficient of friction must = $\tan \theta = 0.70$.

   2) Keeping the surface of the rotating table horizontal, I slowly increased the rotational velocity with the mass at a radius of 0.18 m. At some point, the speed was great enough to cause the mass to slide off the rotating surface with a tangential velocity. The mass fell about 1.08 m to earth and I was able to measure that the mass landed 0.7 m away from where it left the rotating surface.

This is a glorious combination of kinematics (projectile motion) and (circular) dynamics. It requires two drawings, which I expect of you, but I am not supplying right now:

Using dynamics, make a good FBD and follow the protocol and you can see that the force of friction is the only radial force, so you set it equal to mass*centripetal acceleration. This would provide us with the coefficient of friction, except that we don't know the speed… but we know how far it moves horizontally when it falls, so we go to the kinematics lens.

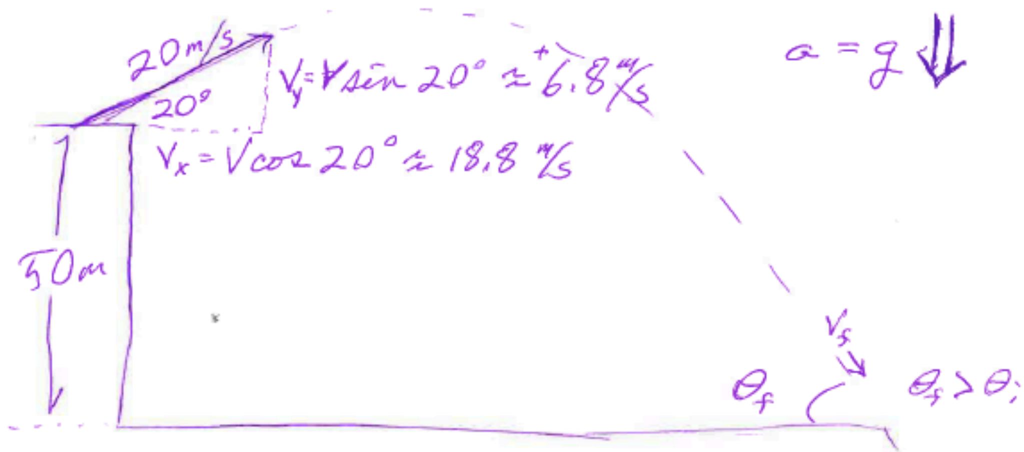
Please show the parabolic trajectory as the coin falls from the edge of the spinning turntable. We want the initial horizontal velocity = $\Delta x/\Delta t$. $\Delta x = 0.7$ m, but how about Δt? This is revealed in the vertical component because the coin is falling from rest (vertically speaking) and hits the ground. I could use energy to find v_{final} and then use $v_{average} = ½v_{final} = \Delta x/\Delta t$. Or I could just use kinematics that $\Delta y = ½at^2$. You should find that it takes about 0.46 s to fall 1 m from rest, so the initial horizontal speed is about 1.5 m/s.
Now we can go back to our circular dynamics lens and see that the centripetal acceleration at this speed at a radius of 0.18 cm is about 12.6 m/s^2, requiring a frictional force of m*12.6 m/s^2. Knowing the vertical acceleration = 0, the normal force = mg; yielding a coefficient of friction of 1.26.

Bummer… we expected the static coefficient of friction to be the same for both scenarios and this discrepancy is outside of what I would have found to be reasonable uncertainty of our measurements. However, we only did the experiment once and the value of speed was rather unsure.

5) Hit a baseball off a cliff: Exercise 6, section 7.6

   There's two ways to solve this that I know if. Strictly kinematics, you can make a good drawing and decompose the initial velocity into vertical and horizontal components. We do this because of dynamics because (Gravitational) force cause acceleration (downward). It is the *time* that connects the vertical situation (the ball goes upward, stops, comes downward, with downward acceleration of gravity) while the in the horizontal direction, the ball moves forward at a constant horizontal speed until it hits the ground. It is TIME that connects the two – the ball only moves horizontal for the same amount of time that it is moving up and down. We solve the vertical (quadratic) equation for time, and substitute it into the horizontal equation for constant speed in the x direction to get the distance the ball goes forward before hitting the ground. Then we can look at the vertical velocity! We use $v_f = v_i + -gt$ to find the final vertical velocity and add this to the horizontal velocity in order to get the final velocity. We use trig to find the angle.

   But, I like energy! First I'd make a good drawing. I would use energy to solve this problem because $E_k + E_g \Rightarrow E_k$. Using this, I find v_f then v_{yf} the time, then angle, then distance.



I'll use an energy law because

$E_k + E_g \Rightarrow E_k$

$\frac{1}{2}mV_i^2 + mgh_i = \frac{1}{2}mV_f^2 + \cancel{mgh_f}^{\,0}$

$V_f^2 = V_i^2 + 2gh = (20\,m/s)^2 + 2(10\,m/s^2)50\,m$

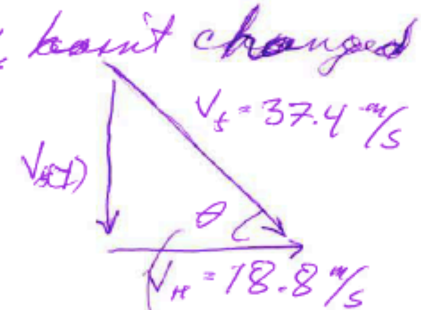
$= 400\,m^2/s^2 + 1000\,m^2/s^2$

$= 1400\,m^2/s^2$

$V_f = (1400\,\frac{m^2}{s^2})^{\frac{1}{2}} = 37.4\,m/s$

reconstructing V_f, we know V_x hasn't changed

$V_H = V_x = 20\,m/s \cos 20° \approx 18.8\,m/s$



using Pythagorus:

$\cancel{V_{fy}}$ $V_{f_y}^2 + V_H^2 = V_f^2$

$\theta = \tan^{-1}\left(\frac{32.4\,m/s}{18.8\,m/s}\right) = 59.9$

or $V_{fy} = \cancel{-41.9\,m/s}\ 32.4\,m/s = V_{iy} + -gt$,

$a = \frac{\Delta V}{\Delta t}$, so $\Delta t = \frac{\Delta V}{a} = \frac{V_f - V_i}{a} = \frac{-\cancel{41.9}\ 32.4\,m/s - 6.8\,m/s}{-10\,m/s^2}$

$= 3.9\,s$

So, the distance traved horizontally is:

$\Delta x = V_{ave} \cdot \Delta t = V_H \cdot \Delta t \approx 18.8\,\text{m/s} \cdot \cancel{4}.9\,\text{s} \approx 91.5\,\text{m}$

Kinematics because we are looking for motion as an explicit function of time.

Using the same drawing, we separate the motion into x, y equations

x, horizontal $a = 0$

$x_f = \cancel{x_i} + V_i t + \frac{1}{2}\cancel{a}t^2$, $V_i = 18.8\,\text{m/s}$

$x_f = 18.8\,\text{m/s} \cdot 3.9\,\text{s} = 73.6\,\text{m}$

$\cancel{= V_i \Delta t}$

$V_f = V_i + \cancel{a}t = V_f = 18.8\,\text{m/s}$

$V_f = ((32.2\,\text{m/s})^2 + (18.8\,\text{m/s})^2)^{\frac{1}{2}}$
$= 37.1\,\text{m/s}$

$\theta = \tan^{-1}\left(\frac{32.2}{18.8}\right)$
$\approx 59.7°$

y, vertical $a = -g$

$y_f = y_i + v_i t - \frac{1}{2}gt^2$, $v_i = 6.8\,\text{m/s}$

$0 = \underbrace{y_i - y_f}_{50\,\text{m} = c} + \underbrace{v_i}_{b} t - \underbrace{\frac{1}{2}g}_{a = -5\,\text{m/s}^2} t^2$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6.8\,\text{m/s} \pm \left((6.8\,\text{m/s})^2 - 4(-5\,\text{m/s}^2)(50\,\text{m})\right)^{\frac{1}{2}}}{2(-5\,\text{m/s}^2)}$

$= .68\,\text{s} \pm 3.24\,\text{s}, -2.6\,\text{s}, \underline{3.9\,\text{s}}$

3.9s

$V_f = V_i + at = 6.8\,\text{m/s} - 10\,\text{m/s}^2(3.9\,\text{s})$
$= 6.8\,\text{m/s} - 39\,\text{m/s}$
$= -32.2\,\text{m/s}$