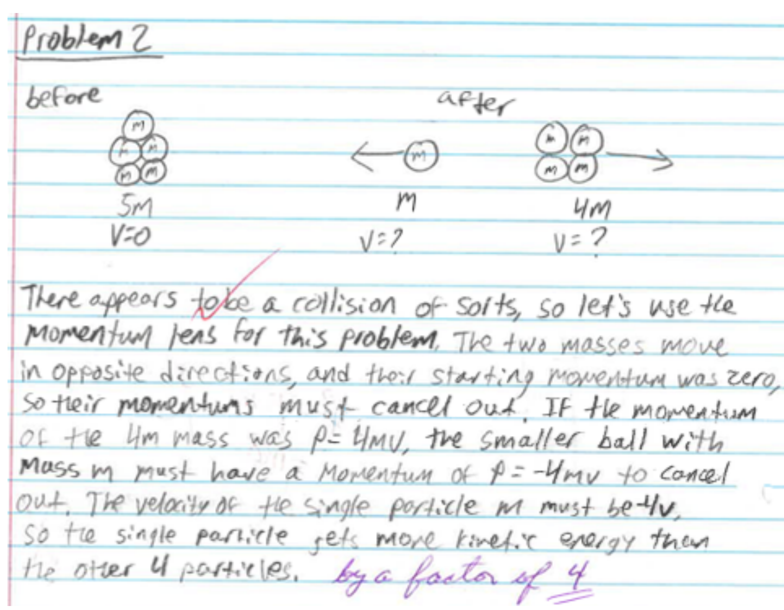
Problem Set #3 due beginning of class, Monday Jan. 28. Please state the lens you are using and why. Remember that you are graded on your ***communication*** of physics understanding.

1. Imagine that you are traveling downward in an elevator at a rate of about 10 m/s, but you are slowing down at a rate of 2 m/s every second. The mass of the elevator is 1000 kg (with you in it). I want to find the tension in the cable holding the elevator.
   a) I bet you already made a drawing and are considering everything involved.
   b) Please consider all the lenses quickly. Choose one and provide the motivation.
   c) If you chose dynamics, why would you do this? I mean, what is your motivation?
   d) What is the complete *mathematical* relationship between forces and acceleration that define dynamics?
   e) If you haven't done it, identify these forces with a free body diagram!
   f) Why is it very (very very) important to identify the direction of acceleration in a FBD?
   g) Between the tension and the force of gravity, which force is larger or are they the same? Why can you be sure?
   h) With a forces diagram, show how you add the forces on the elevator to find the resultant force.
   i) Find the tension of the cable from which the elevator is suspended.

   Please see solutions from BE!#3

2. *From an old midterm. Even if you've never heard of fusion, you have the basic skills to draw a picture and analyze this problem.* Fusion is the process that powers the sun and hydrogen bombs: small nuclei are fused into larger nuclei. One fusion process involves a triton (two neutrons and a proton – recall that neutrons and protons have about the same mass) and a deuteron (one neutron and a proton) fusing to form a supercharged 5-nucleon nucleus, which gives off its energy by blasting apart into a single neutron and a helium nucleus (or alpha particle) at high speeds. I want to know which of the particles gets more of the energy. Let's simplify the problem to just the explosive breakup: Protons and neutrons have the same mass, so we can think of this process as **a 5-ball cluster (in space, at rest) breaking up through an energetic explosion into one ball and a 4-ball cluster. Do the two pieces equally share the kinetic energy or does one get all or more kinetic energy?** You will be graded not on your answer, but on your reasons, drawings, and lens descriptions.
   a) Make a good drawing of this process… maybe two drawings (before and after).
   b) One of your friends announces, "the energies must be equal because energy is conserved!" Please help this person out. What does conservation mean (and doesn't mean)? You can always refer them to section 1.8 in our text.
   c) Now, do your best to consider the process and what might be conserved and why. Consider what changes and how this will affect the ratio of speeds and energy.

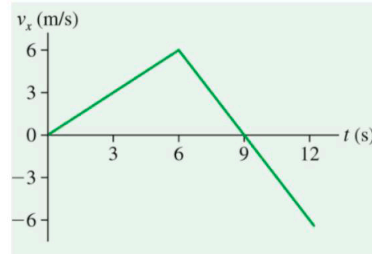
We need to start with an energy lens, recognizing that the total kinetic energy of the two particles comes from the explosion… from the nuclear energy given off. However, we don't know how it is divided. But we can use a force or momentum lens. We know that $\boxed{\Delta\vec{p} = \vec{F}\Delta t}$, and there are no outside forces so the system's momentum is conserved. It starts at zero and must then is zero after the explosion… this is like the "pushing off the boat" problem! *Hence the two clusters must have equal and opposite momenta in order to have a sum momenta of zero. From there, please show that the single neutron must have 4 times the speed as the 4-ball cluster, in comparing the kinetic energies, the neutron should take 80% of the energy of the explosion, or 4 times that of the larger cluster.*



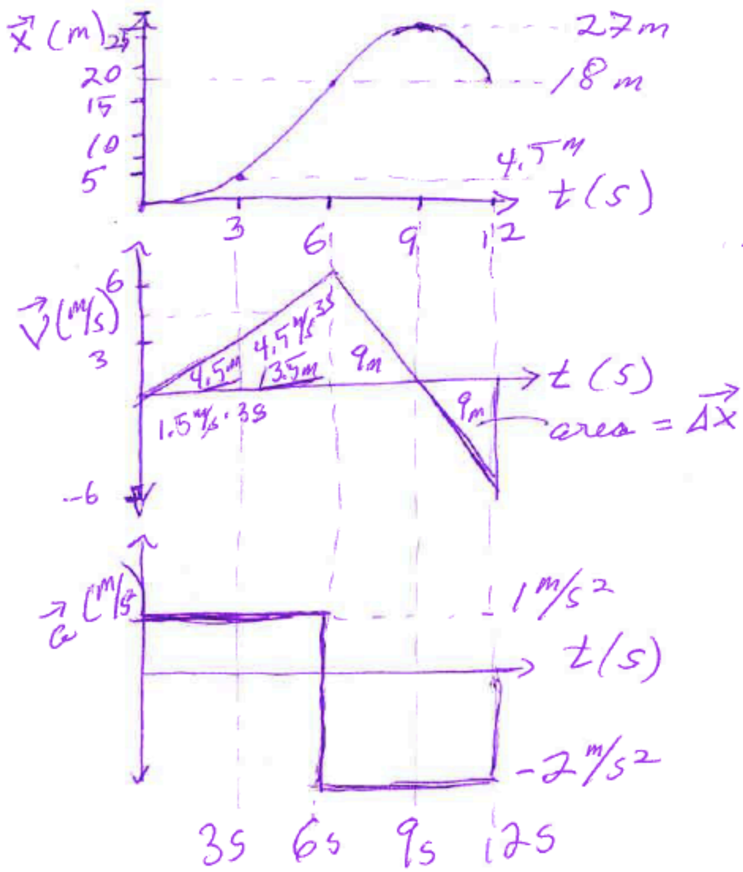
There appears to be a collision of sorts, so let's use the momentum lens for this problem. The two masses move in opposite directions, and their starting momentum was zero, so their momentums must cancel out. If the momentum of the 4m mass was p=4mv, the smaller ball with mass m must have a momentum of p=-4mv to cancel out. The velocity of the single particle m must be 4v, so the single particle gets more kinetic energy than the other 4 particles. by a factor of 4

3. Please see the velocity time graph at right for an object that starts at $x = -10$ m. Please:

   a) Write a narrative – what is happening in the graph.
   b) Make an acceleration vs time graph.
   c) Make a position vs time graph.



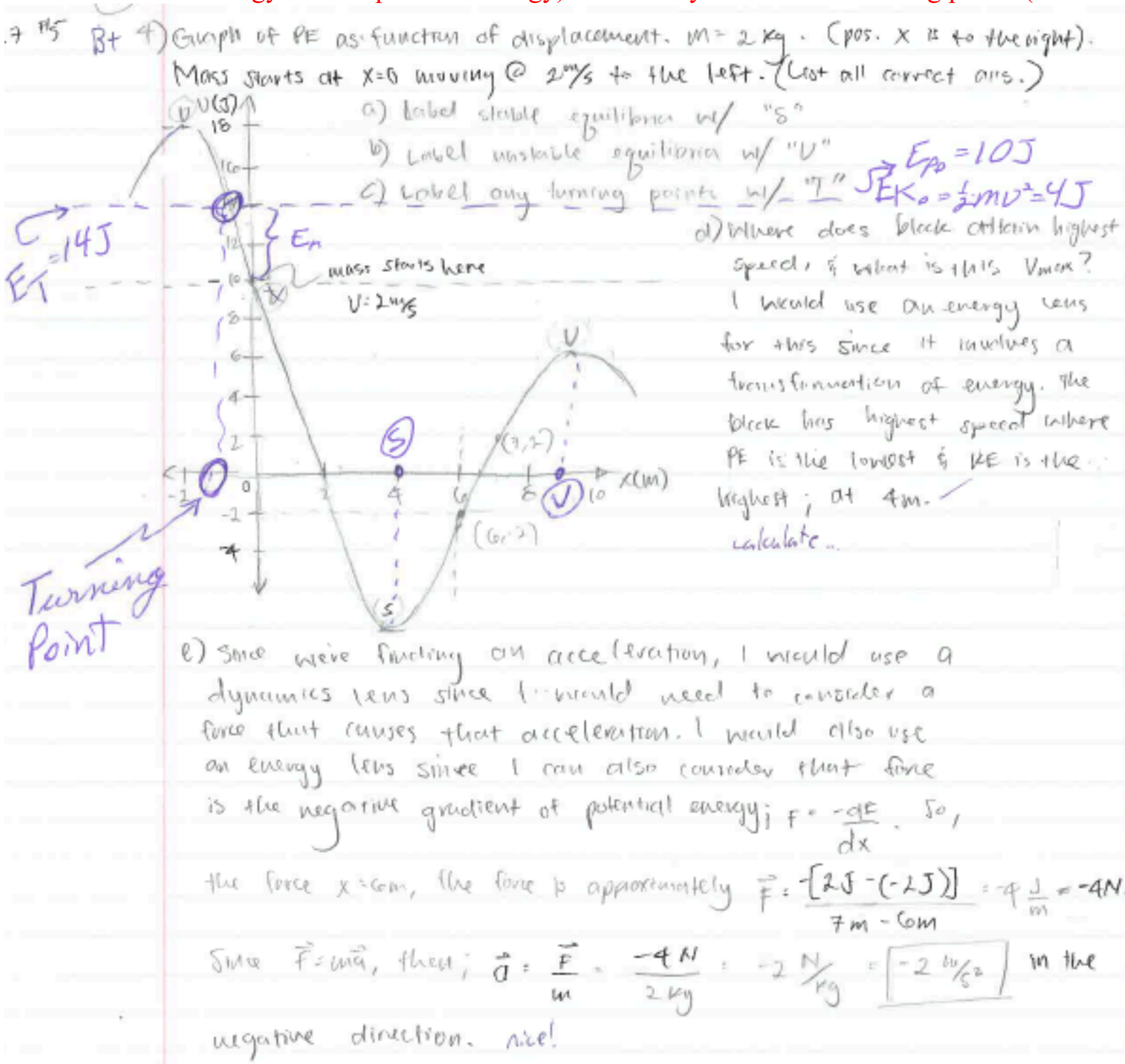
My friend and I are in a canoe, starting from rest, we begin paddling, causing the canoe to accelerate at 1 m/s^2. We do this until we get up to a nice speed of 6 m/s after 6 s. During this time, our average speed was 3 m/s, so we moved a total of 18 m forward… when a large crocodile surfaces in front of us, and we quickly begin paddling backwards. It takes us 3 seconds to come to reset… during this time, we move an additional 9 meters forward, but we begin moving backwards and speed up in 3 seconds to be moving at 6 m/s backwards, over which time, we backtracked the 9 m it took us to slow to a stop.



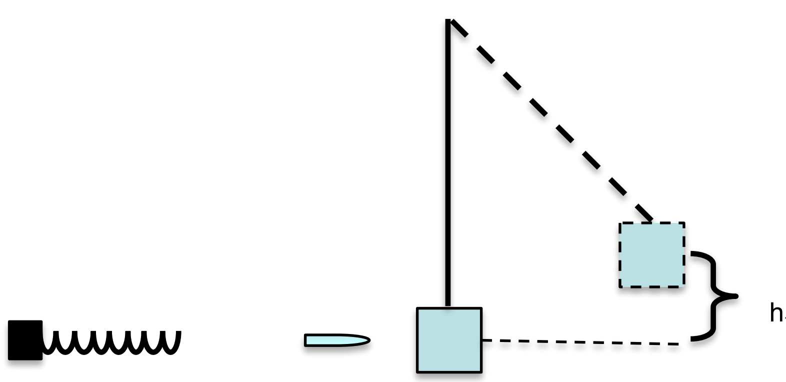
max $\vec{\Delta x}$ = 4.5m + 13.5m + 9m
= 27m

4. Exercise 5 in 2.7, potential energy graph. Traditionally, students have a hard time with this. Please consider reading through 2.7 while you do this example and/or watching the associated video.

The first thing you want to do with potential energy graphs is find the total energy = $E_P + E_K$. Draw this line in (as you see the student below did). This will give you the kinetic energy at all points (the difference between the total energy and the potential energy) and allow you to find the turning points (where $E_k = 0$).



5. A loaded gun is cocked by compressing a spring of k = 10^4 N/m. and then releasing it behind a 20 g bullet. The bullet strikes and sticks inside of a 0.5 kg ballistics pendulum and swings upward to a final height of 50 cm. Presume the spring is massless and there is no friction in the system. We want to find everything: The bullet's speed; how far the spring was originally compressed; The maximum acceleration of the bullet in the gun.



a) Your friend announces, "I'm using an energy lens to find *everything* because energy is always conserved!" Please explain to this friend that while energy is always conserved, it does convert in sneaky ways, and thus may not be universally usable… In particular, what happens here that may render the energy lens ineffective?

b) In fact, you will need to use two different lenses to solve this problem… maybe more depending on how you solve this, but two in particular are effective. Please describe the process from the moment I let go of the spring to the moment the pendulum swings to its highest point. In your description, for each event, identify a lens and state why you are using that lens.
c) Please calculate the speed of the bullet before it hits the pendulum.
d) Please calculate how far the spring was compressed.
e) Does the bullet have constant acceleration in the gun, or does the acceleration change over time? Please explain your answer… identify a lens.
f) Please find the maximum acceleration of the bullet in the gun.

As soon as we see this, we are tempted to use an energy lens equating the initial spring potential energy to the final gravitational potential energy. However, the great majority of the bullet's kinetic energy is converted to thermal energy in the inelastic collision. Thus, we can find the kinetic energy of the bullet/block immediately after collision using an energy lens. However, to find the bullet's speed, we need to use the momentum lens because there is negligible outside forces so the momentum is conserved in the collision. The bullet's kinetic energy does come from the spring potential energy.
For letter "c" and "d", constant acceleration would be the result of a constant force. However, the force of the spring is proportional to the spring's compression. This the maximum acceleration would be when the spring is maximally compressed. This acceleration comes out to be 6000 times the acceleration of gravity, but so is the life a bullet! In fact, this acceleration is small compared to the acceleration the bullet experiences when it hits the target!

6). Lens: Energy Lens because energy is being converted from potential to kinetic and momentum

$k = 10^4$ N
$m_b = .02$ kg
$m_{box} = .5$ kg
$h_f = .5$ m

$E_{spring} \longrightarrow KE_{Bullet} \longrightarrow E_{therm} + KE_{Block} \longrightarrow PE_g$

$\frac{1}{2}kx^2 \longrightarrow \frac{1}{2}mv^2 \longrightarrow \quad \longrightarrow mgh$

$\frac{1}{2}(10000\,\frac{N}{m})x^2 \xrightarrow{\text{units}} \frac{1}{2}(.02\,kg)v^2 \quad \longrightarrow (.5\,kg)(.5\,m)(10\,m/s^2)$

inelastic collision of bullet / box

lens? $\vec{p}$ because $F_{outside} = 0$

$\frac{1}{2}mv^2 \longrightarrow 2.6\,J$

$\frac{1}{2}(.52\,kg)v^2 = 2.6\,J$

$v = \sqrt{\frac{5.2\,J}{.52\,kg}}$

$v_{box} = 3.16$ m/s

$m_b v_b = m_{box} v_{box}$

$(.02\,kg)v_b = (.52\,kg)(3.16\,m/s)$

$v_{bullet} = \frac{(.52\,kg)(3.16\,m/s)}{(.02\,kg)} = \boxed{82.16\ m/s}$ a

$\frac{1}{2}(10000\,\frac{N}{m})x^2 = \frac{1}{2}(.02\,kg)(82.16\,m/s)^2$

Energy lens, because $E_s \Rightarrow E_K$

$x = \sqrt{\frac{(.02\,kg)(82.16\,m/s)^2}{10000\,N/m}} = \boxed{0.12\ m}$ b

$\left(\frac{kg\,\frac{m^2}{s^2}}{N/m}\right)^{\frac{1}{2}} = (m^2)^{\frac{1}{2}} = m$

c) The force of the bullet comes from the force of the spring ($F = kx$). To have a constant acceleration, there must be a constant force. The force of the spring is not constant because as the ~~displacement of compression change.~~ spring reaches equilibrium, the F_{spring} decreases. Thus the bullet does not have constant a.

Please include units.

Nice!

d) $F_s = F_B = ma$

$\frac{1}{2}(10000)(.12)^2 = (.02)a$ $\quad \boxed{a = 58000\ m/s^2}$

E_p

$F = kx = 10000\,\frac{N}{m} \cdot 0.12\,m = 1200\,N!$

e) Lenses are the way to go!

Yes!

$a = \frac{F}{m} = \frac{1200\,N}{0.02\,kg} = 60000\,\frac{kg\,m/s^2}{kg}$

$= 6 \times 10^4\,m/s^2 = 6000\,\text{gravities}!$

6. Using an energy lens, please show that if you drop a 5 kg box from 60 m, it hits the ground at ~35 m/s. Then we *throw* the box *downward* from 60 meters height with an *initial speed* of 35 m/s.
a) Dropping a box from 60 m, find the speed when it hits the ground.
b) Throwing the box *downward* at 35m/s from a 60 m cliff, find the speed that it has when it hits the ground.
c) What if I throw the box *upwards* at 35 m/s, what is the speed when it hits the ground?
d) What if I throw it straight off the cliff at 35 m/s horizontally, what speed does it have when it hits the ground now?
e) Can I throw a 5 kg box at 35 m/s? Please back up your answer.
I show in a video that if I double the energy, then the speed increases by root 2 or about 49 m/s. Also, conserving energy, it doesn't matter what angle I throw the box, the final kinetic energy (and speed) will be the same. We also see, using an energy lens, it is very unlikely I could throw 5 kg at that speed, requiring power and force from my arm that is really more than one would expect from me.

7. According to the hydrodynamic flow equations you'll learn in PHYS-122, the speed of water coming from a 200 PSI fire house is about 45 m/s (~100 mph!). Wikipedia claims these hoses are 25 mm in diameter. Imagine if you were hit with water by one of these hoses, like if you were protesting the Dakota Access Pipeline, and the fire department was called to clear the area (please see some drama: https://www.youtube.com/watch?v=K3Iv9okL4QU). I'd like to know the force that this water puts on someone's body. Let's model the water as a moving column that hits you and disperses all directions perpendicular to its original direction of travel, as in the figure of the demonstrator at right.

a) In order to solve this problem, I suggest you use a force/momentum lens. Please describe what happens with momentum in this problem, and how this consideration would lead to a calculation of force on her body.
b) Imagine a section of water headed toward her, 25 mm in diameter and 1 meter long. Calculate the volume of this column, the mass of water contained, and the column's momentum before hitting her.
c) What is the total momentum of water *after* it hits her body according to our model above?
d) How long did it take the water to change momentum?
e) Find the force that this water puts on her body. Estimate a reasonable acceleration with this force. Could it knock her over?

Please see solution in firehose video

8. We clocked Harrison's throwing speed with our ballistics pendulum. The mass of the ball was 41 g, the mass of the cooler that he threw the ball into was 2.0 kg. The cooler swung back gaining about 1 cm of elevation.
a) Set out a plan for calculating the throwing speed.
b) Calculate the throwing speed.
c) Find out how much kinetic energy was changed to heat in the collision when the ball hit the cooler.

As we discussed in class, this problem requires the use of two lenses. We'd like to use an energy lens because the kinetic energy of the bag turns to kinetic energy of the bag and pendulum, turns to gravitational potential energy of the bag and pendulum. However, in the inelastic collision, an unknown amount of kinetic energy is "lost" in the form of thermal energy. So, we need to look at the collision through a momentum lens. Please solve this problem backwards in time. The gravitational potential energy of the pendulum came from the initial kinetic energy it had immediately after the collision:

$E_K \Rightarrow E_g$

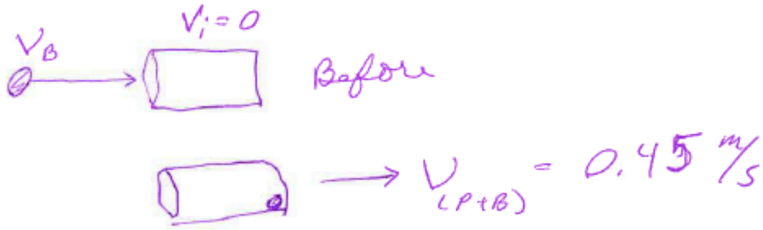
$$\tfrac{1}{2}\underline{m_{(P+B)}}\,v^2_{(P+B)} = \underline{\overset{m_{(P+B)}}{m}}\,g\,\Delta h$$

masses are same + cancel

$$v = (2g\Delta h)^{\frac{1}{2}} = (2\cdot 10\,\tfrac{m}{s^2}\cdot 0.01\,m)^{\frac{1}{2}} = (0.2\,\tfrac{m^2}{s^2})^{\frac{1}{2}}$$
$$\approx 0.45\,\tfrac{m}{s}$$

That's ~ $\frac{1}{2}$ m/s or 1 mph ...
but it wasn't moving very fast after the collision.

Then we recognize that during the collision, there was negligible outside forces, so the change in momentum of the system should be zero. Thus, we can conserve momentum in the collision.



$$\Delta \vec{P}_s = 0$$

$$\Sigma \vec{P}_i = \Sigma \vec{P}_f$$

$$\vec{P}_{B_i} + \vec{P}_{P_i} = \vec{P}_{(B+P)_f}$$

$$m_B V_B + m_P \overset{0}{\cancel{V_P}} = m_{(B+P)} V_f$$

$$V_B = \frac{m_{(B+P)} V_f}{m_B} = \underbrace{\left(\frac{2041\,g}{41\,g}\right)}_{=\sim 50} 0.45 \text{ m/s} \approx \underline{22.4 \text{ m/s}}$$

Essentially, we can say that the mass of the moving body increased by a factor of 50, so the speed must have decreased by a factor of 50… so what about the kinetic energy? Because velocity is squared, we see that the kinetic energy decreased by a factor of 50… or we lost 49/50 (98%) of the kinetic energy of the ball in the collision:

in the collision $m \Rightarrow 50\, \underline{m_i}$ so $V \Rightarrow \frac{1}{50} V_i$

so $E_K \Rightarrow \frac{1}{2} m V^2$

$= \frac{1}{50} E_{K_i}$

Please carry out the calculation, and show that the original kinetic energy of the ball was ~ 10.3 J, and the kinetic energy of the pendulum and ball system was about 0.21 J.