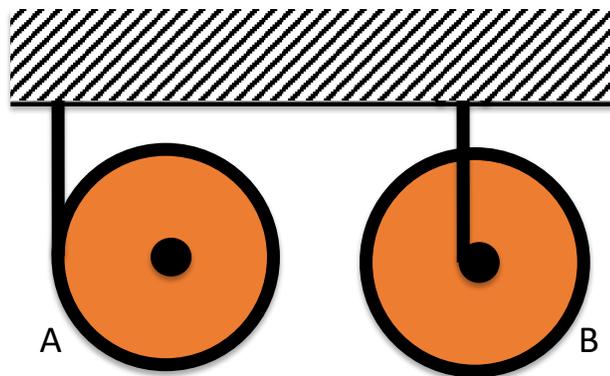


You will be graded on your communication of physics understanding.

#1 At right you see two identical wheels that are simultaneously let go. They are both connected to the same length of string, but the string is wrapped around the rim of wheel A, and the string is wrapped about the small center hub of wheel B.



- a) Which wheel is spinning the fastest (highest rotational velocity) when it comes to the end of the string, or are they the same? Please outline your argument very clearly.
- b) The string connected to which wheel has greater tension, or do they have the same tension? Please be very clear in supporting your answer.

Because energy is conserved, the wheel that is spinning fastest at the end, must be moving the slowest (linearly). However, it is NOT correct to assume that the mass with the largest acceleration has the lowest rotational acceleration. In fact, looking at torques, please show that wheel A has the highest rotational AND linear acceleration... and arrives first at the ground. Wheel B ultimately gains more rotational energy than A, but it takes a long time!

supporting your answer.

a) I'm going to use an energy lens since there are energy conversions and energy is conserved. Energy changes from $PE_g \rightarrow KE + RE$. We know both wheels start at the same height so their potential energies are the same.

$$PE_A = PE_B$$

$$KE_A + RE_A = KE_B + RE_B$$

$$\frac{1}{2}mv_a^2 + \frac{1}{2}I\omega_a^2 = \frac{1}{2}mv_b^2 + \frac{1}{2}I\omega_b^2$$

$$\frac{1}{2}m(R\omega_a)^2 + \frac{1}{2}I\omega_a^2 = \frac{1}{2}m(r\omega_b)^2 + \frac{1}{2}I\omega_b^2$$

$$R\omega_a^2 + \omega_a^2 = r\omega_b^2 + \omega_b^2$$

$$\omega_a^2(R+1) = \omega_b^2(r+1)$$

$$\omega_a^2 = \omega_b^2 \left(\frac{r+1}{R+1}\right) \rightarrow \left(\frac{r+1}{R+1}\right) < 1$$

$I = \frac{1}{2}mR^2$

We know $v_{cm} = r\omega$ plus in for unknown v

Since radius about center of rotation for A is greater than radius B, their velocities ($v=r\omega$) would differ.

~~That's great!~~
That's great!

In order for their potential energies to be the same, the sum of their KE and RE must be the same. Since their radii about the axis of rotation differ - big R for A and little for B - their linear velocities would differ. The linear KE for A is greater than KE for B. Simultaneously, and for energy to be conserved, the RE for A must be less than RE for B. Therefore, wheel B is spinning faster than wheel A.

b) The force of tension and gravity are causing acceleration, so I will use a dynamics lens. Since the linear kinetic energy is greater for A than B, its expected velocities and accelerations would follow the same pattern. We know the acceleration for wheel A will be greater than the linear acceleration of B.

Wheel A

Wheel B

$$\sum \vec{F} = m\vec{a}$$

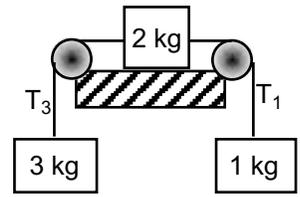
$$T - F_{tA} = m\vec{a}_A$$

$$T - F_{tB} = m\vec{a}_B$$

Gravity and mass are constant, and the \vec{a}_A is greater than \vec{a}_B , therefore, Force of Tension A must be greater than the force of tension B.

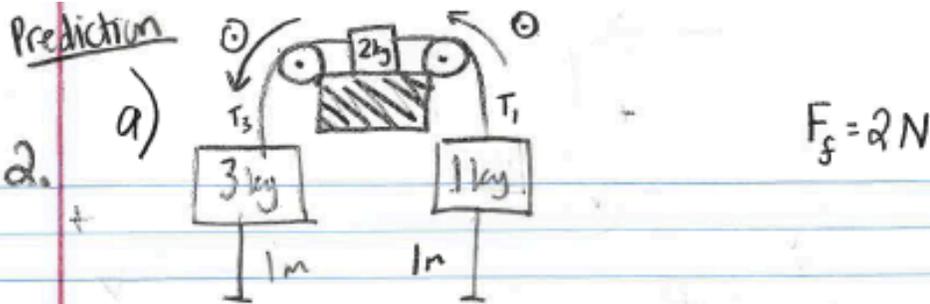
#2 Check out the system at right. The two hanging masses are 1 m from the floor.

The force of friction between the 2 kg mass and the surface is 2 N. The string slides with almost no friction over the two wheels shown.



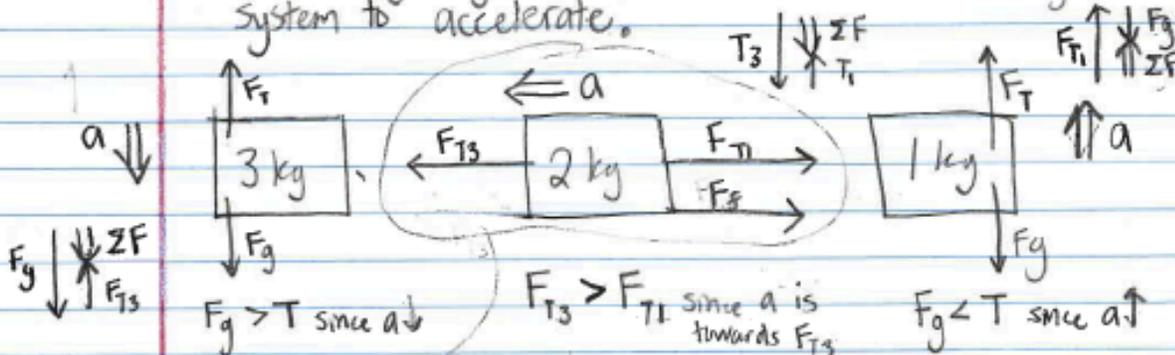
- Make a physically-correct statement about what will happen when I let this system go from rest. Include relevant direction.
- Compare T_3 to T_1 . Provide good reasoning for your answer.
- Find either the acceleration of the system after I let it go, or find the speed of the system just before one of the masses hits the ground.

Folks are very good at comparing (for instance) T_3 to F_{g3} . We know that we have to look through a dynamics lens at the 3 kg mass because these two forces act on the 3 kg mass, noting that its acceleration is downward! Thus, $T_3 < F_{g3}$. So, when we want to compare T_3 to T_1 , which mass should we look at? Many people found the acceleration of the system to be 3 m/s^2 , and from that actually calculated the tension. However, there's an easy way... which way is the 2 kg mass accelerating? What's causing this acceleration?



Dynamics Lens

Forces of gravity, tension, and friction are causing the system to accelerate.



b) When comparing T_1 and T_3 , I know that $T_3 > T_1$ because the 2 kg is accelerating to the left.

$$\Sigma F_{\text{system}} = m_{\text{system}} a = (6 \text{ kg})(a)$$

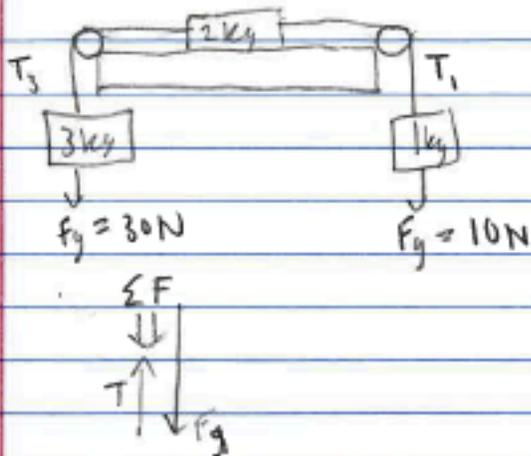
$$a = \frac{\Sigma F}{m} = \frac{F_{T1} - F_{g(1\text{kg})} - F_f + F_{T3(2\text{kg})} - F_{T1(2\text{kg})} + F_{g(3\text{kg})} - F_{T3}}{1 \text{ kg} + 2 \text{ kg} + 3 \text{ kg}}$$

$$a = \frac{-F_{g(1\text{kg})} - F_f + F_{g(3\text{kg})}}{6 \text{ kg}}$$

$$a = \frac{-(1\text{kg})(10\text{m/s}^2) - 2\text{N} + (3\text{kg})(10\text{m/s}^2)}{6 \text{ kg}}$$

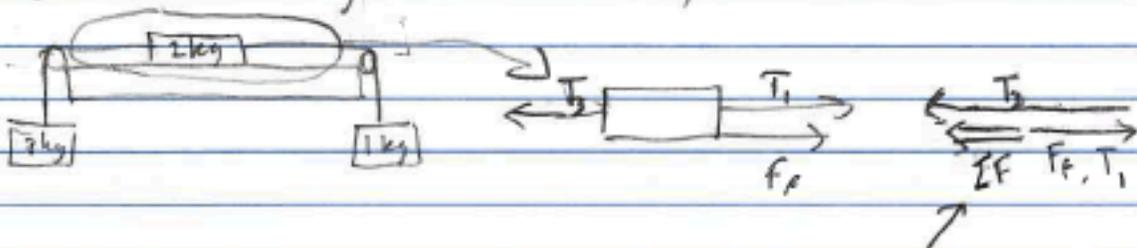
$$a = 3 \text{ m/s}^2$$

a) We can use a dynamics lens b/c the force of gravity and force of tension is causing an acceleration



When you let go of the system, it will accelerate in the downward direction of the left side and upward direction on the right b/c $30N > 10N$

b) We can use a dynamics lens for the same reason in part (a). However, to solve this problem we have to focus on the 2kg mass in the system.



We can see that $T_3 > T_1$
b/c the net acceleration is in the direction of T_3

(c) We can use a dynamics lens b/c the forces of gravity and friction are causing an acceleration. We can use the $\Sigma F = ma$ equation to solve this equation to consider all the forces acting on the system

$$F_{gA} + F_r + F_{gB} = (m_A + m_B + m_C) \vec{a}$$

$$30N - 2N - 10N = (3kg + 2kg + 1kg) \vec{a}$$

$$18N = (6kg) \vec{a}$$

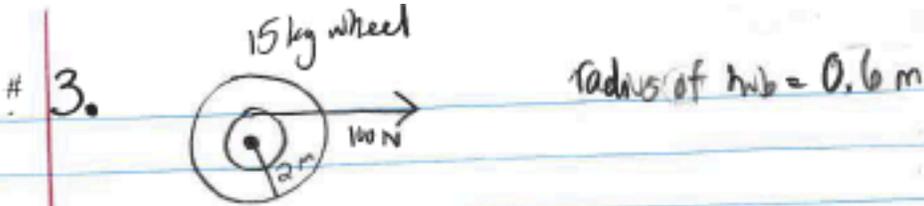
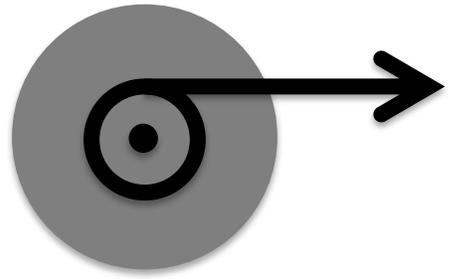
$$\vec{a} = 3m/s^2$$

A

nice!

#3 At right, you see a 15 kg wheel of radius 2 m. It has a central round hub of radius 60 cm. I wrap a string around the central hub and pull with a tension of 100 N.

- a) What is the angular acceleration of the wheel? Many folks mix up Tension and Torque. Are these the same things? What are the units? How do we use them? Folks are also mixing up acceleration and angular acceleration.
- b) How long does it take for the wheel to be spinning at 3 radians/s?



Angular Dynamics

The force Pete exerts on the system creates a torque on the wheel, causing for the wheel to angularly accelerate.

$$\tau_{\text{hub}} = F_{\text{Pete wheel}} r = (100 \text{ N})(0.6 \text{ m}) = 60 \text{ Nm}$$

$$\tau_{\text{hub}} = I_{\text{wheel}} \alpha$$

$$60 \text{ Nm} = \left(\frac{1}{2}\right)(15 \text{ kg})(2 \text{ m})^2 \alpha$$

$$\alpha_{\text{wheel}} = 2 \text{ rad/s}^2$$

Kinematics

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

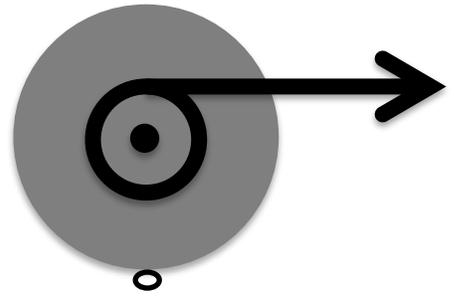
$$\omega = 3 \text{ rad/s}$$

$$2 \text{ rad/s}^2 = \frac{3 \text{ rad/s}}{\Delta t}$$

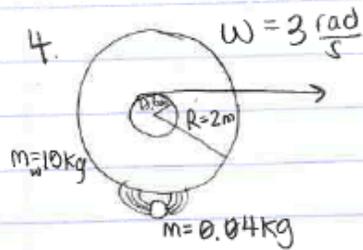
A

$$\Delta t = 1.5 \text{ sec}$$

#4 In the last problem, A large tarantula of mass 40 g is tightly clutching the outside of the rim as shown at the bottom of the ($r = 2\text{m}$, 15 kg) wheel. What is the force that the legs provide to keep the bug on the rim at the rotational velocity of $3/\text{s}$ when in the position shown at right? Clearly explain your answer. Again, many people use rotational and linear mechanics (especially dynamics) interchangeably. They are not the same... they even have different units.



Many people referred to a normal force... is the surface of the wheel pushing onto the spider here? Or are the legs pulling (under tension) on the spider?



- i. lens: Dynamics & Rotational Dynamics
- ii. The force between the tarantula's legs and the wheel cause a_c to keep the spider on the wheel.

$$a_c = \frac{v_L^2}{r}$$

$$\omega = \frac{v_L}{r}$$

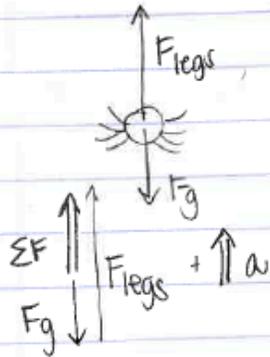
$$3 \text{ s}^{-1} = \frac{v_L}{2 \text{ m}}$$

$$a_c = \frac{(6 \text{ m/s})^2}{2 \text{ m}}$$

$$v_L = 6 \text{ m/s}$$

$$a_c = \frac{36 \text{ m}^2/\text{s}^2}{2 \text{ m}}$$

$$a_c = 18 \text{ m/s}^2$$



$$\Sigma F = m \vec{a}$$

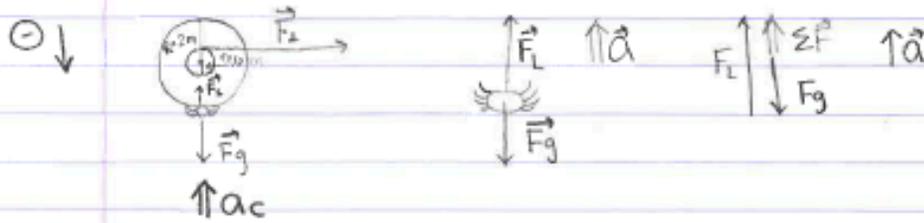
$$-F_g + F_{\text{legs}} = 0.04 \text{ kg} (18 \text{ m/s}^2)$$

$$-(10 \text{ m/s}^2)(0.04 \text{ kg}) + F_{\text{legs}} = 0.72 \text{ kg} \cdot \text{m/s}^2$$

$$F_{\text{legs}} = 0.72 \text{ kg} \cdot \text{m/s}^2 + 0.4 \text{ kg} \cdot \text{m/s}^2$$

$$F_{\text{legs}} = 1.12 \text{ N}$$

#4 Lens = dynamics because
 Σ Forces causes acceleration



$$\Sigma \vec{F} = m\vec{a}_c \quad \vec{F}_{\text{legs}} - \vec{F}_g = m\vec{a}_c$$

$$\vec{a}_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R}$$

$$\vec{a}_c = \frac{(3\text{s}^{-1} \cdot 2\text{m})^2}{2\text{m}}$$

$$\vec{a}_c = \frac{(6\text{m/s})^2}{2\text{m}} = \frac{36}{2} = 18 \frac{\text{m}}{\text{s}^2}$$

$$\vec{F}_{\text{legs}} = m\vec{a}_c + \vec{F}_g$$

$$F_L = m\vec{a}_c + m(10 \frac{\text{m}}{\text{s}^2})$$

$$F_L = (.04\text{kg})(18 \frac{\text{m}}{\text{s}^2}) + (.04\text{kg})(10 \frac{\text{m}}{\text{s}^2})$$

$$F_L = .72\text{N} + .4\text{N} = \boxed{1.12\text{N}}$$

Because the spider is rotating with the disk on the edge, it has centripetal acceleration this is the acceleration the spider has the whole time, this acceleration is upward when the spider is at the very bottom because the a_c goes towards the radius. Then we acknowledge the forces acting on the spider. The only forces are the force of gravity pulling downward on the spider & the force of the spider's legs which is opposite ^{direction} of gravity when it's on the bottom, because the legs are holding the spider to the disk.

A

nice ↗

Name _____

By signing below, I testify that I understand everything I wrote on this exam and could reproduce this work if asked to do the problem again.

Signature: _____