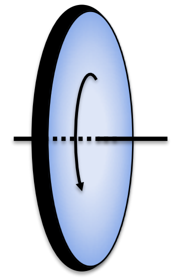
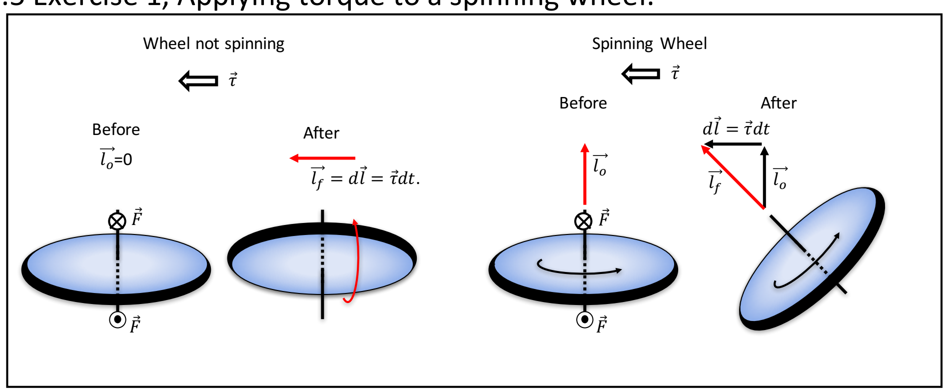
PS#9 Due in Class Thursday, March 14. Please pay good attention to describe the lens you are using and explain your method.

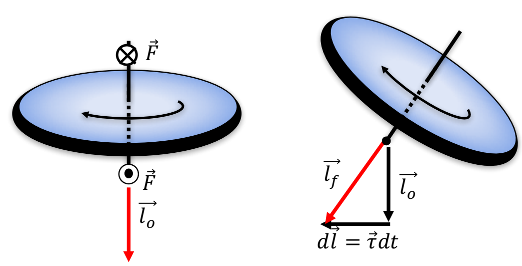


1. You are holding the axle of a bicycle wheel (one hand on each side) out in front of you, spinning as shown.
   a) What is the direction of the angular momentum vector?
   Right Hand Rule: to the right →
   b) You push away with your right hand and pull in with your left hand. What is the direction of the torque you put on the wheel? What is the direction of the angular impulse that you give to the wheel?
   Again, using the right hand rule, this is an upward torque, inducing rotation in the upward direction.
   c) After you push for a moment, how does the orientation of the wheel change?
   The upward torque provides some change in angular momentum in the upward direction or, $d\vec{l} = \vec{\tau}dt$. You add this $d\vec{l}$ to the wheel's previous angular momentum and find the new (resultant) angular momentum. This is still to the right, but slightly upward. Hence, you have rotated the angular momentum vector (and rotated the spinning wheel) counter clockwise, or (by the right hand rule) out of the paper at you.

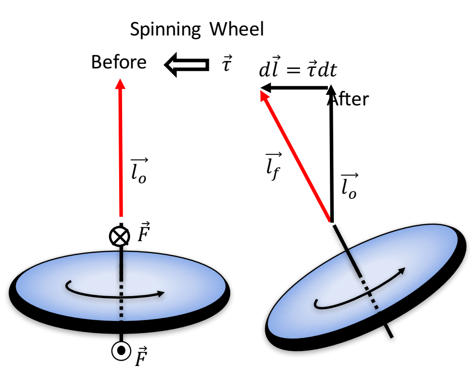
2. 7.5 Exercise 1, Applying torque to a spinning wheel.



- Can you predict the direction the axle will turn? You really should do this with a wheel and we did it.
- Predict what happens when the wheel is spinning in the opposite direction? Why?



If the wheel is spinning the opposite direction, the torque and change in angular momentum are the same, but the initial angular momentum is in the opposite direction. Thus, we see that the wheel will tip in the opposite direction. Thus it will precess in the opposite direction.

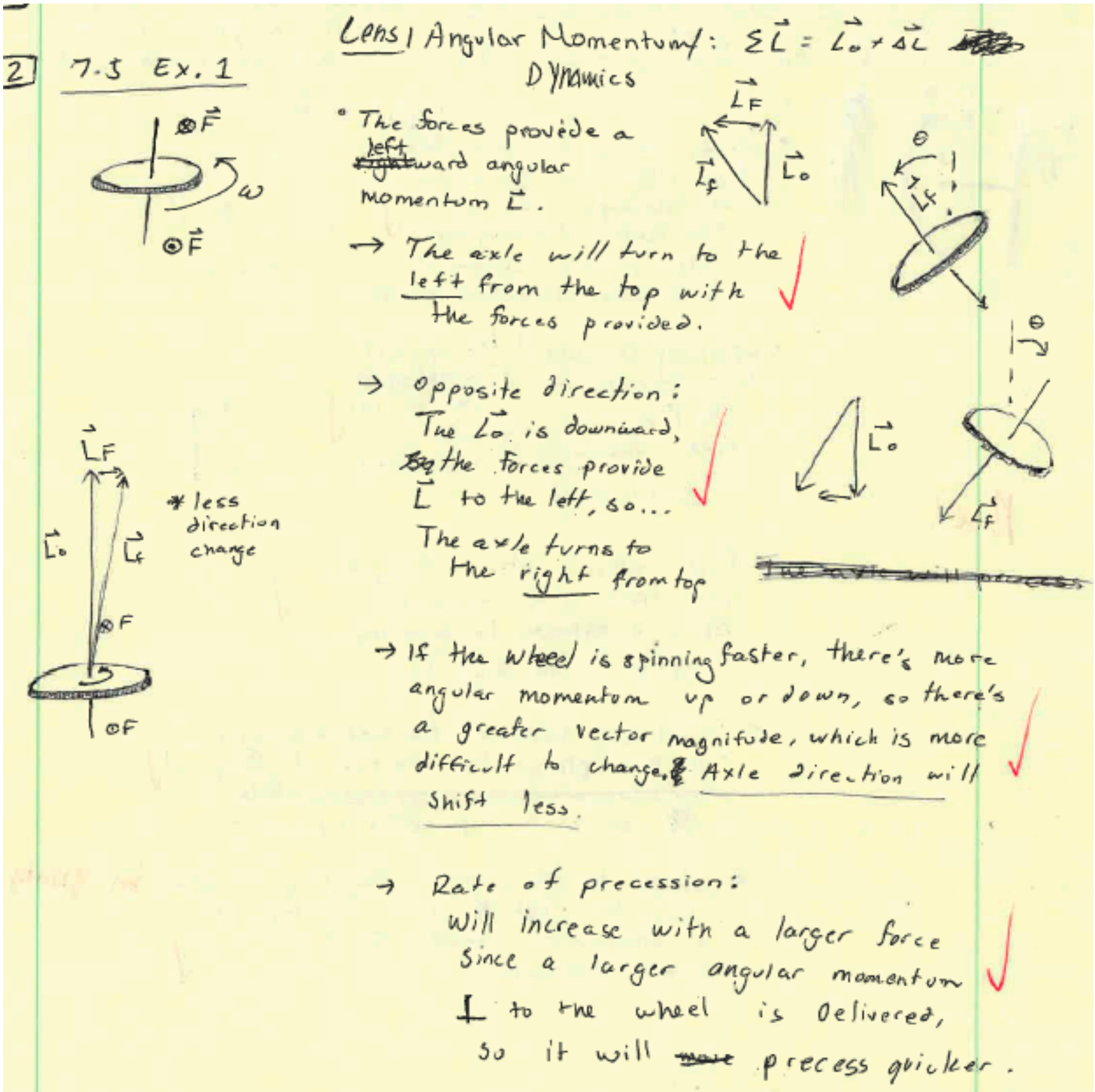


- What happens if the wheel is spinning faster? Can you explain why?

If the initial angular momentum is greater, then change of angular momentum from the same torque of gravity will result in a smaller change of angle of the spinning wheel. So it would precess slower.

- How does the rate of precession change when you push harder on the axle? Why?

If you push harder on the wheel the torque and change in angular momentum will be greater. In the same amount of time, the wheel will tilt by more. Thus, the rate of precession will increase.



2 7.5 Ex. 1

Lens | Angular Momentum/ Dynamics: $\Sigma\vec{L} = \vec{L}_o + \Delta\vec{L}$

- The forces provide a ~~rightward~~ left-ward angular momentum $\vec{L}$.

→ The axle will turn to the left from the top with the forces provided.

→ Opposite direction: The $\vec{L}_o$ is downward, ~~So~~ the forces provide $\vec{L}$ to the left, so... The axle turns to the right from top ~~The axle will precess~~

→ If the wheel is spinning faster, there's more angular momentum up or down, so there's a greater vector magnitude, which is more difficult to change. & Axle direction will shift less.

→ Rate of precession: Will increase with a larger force since a larger angular momentum $\vec{L}$ to the wheel is delivered, so it will ~~move~~ precess quicker.

3. 7.5 Exercise 3 These questions are similar to those in the above questions. However, there are two exceptions:

- What changes if you switch sides and support the axle on the other side? Why?

Supporting the wheel from the other side will reverse the torque that gravity provides. This will result in the wheel precessing in the opposite direction. Please prove this to yourself with a drawing.

- What happens if you support the axle closer to the center of the wheel? Why?

Supporting the wheel close to the axle will reduce the torque from gravity. This will reduce the change in angular momentum, so the rate of precession will decrease.

Angular Momentum / Rotational Dynamics

$\Delta\vec{L}$ is 0 so $\sum \vec{L}_{Total} = \vec{L}_o + \Delta\vec{L}$

- Forces applied at a radius create torques, (impulse on angular momentum) $\vec{\tau} = \frac{d\vec{L}}{dt}$

- Supporting it on the ~~left~~ right side creates a torque and $\vec{L}$ into the paper. $\vec{L}_o$ is to the left. Summing these like vectors shows it will spin direction upward.

- Spinning the wheel the opposite way creates a $\vec{L}_o$ to the right. the ↑ forward sums with the downward $\vec{L}_o$, so spins downward.

Nice!

- Faster spin = slower precession since there is a larger $\vec{L}_o$, so $\Delta\vec{L}$ will be more minimal compared to the large $\vec{L}_o$

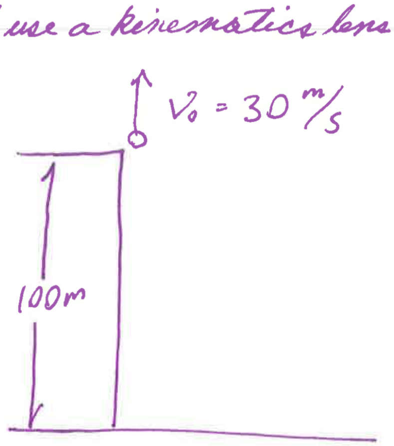
- Switching sides of the supporting axle, it will change the direction of $\vec{\tau}$ applied ~~so it will precess opposite~~ so it will precess opposite

- Closer to the center, the torque applied by gravity will be less since $\tau = F_\perp(r)$, so a smaller r = smaller τ. This means slower precession.

4. 7.6 Exercises 1 and 2, deriving our two kinematic equations. These are covered in the videos, and you don't have to hand them in, but it's a good exercise to do them in order to know where the formulas come from.

5. 7.6 Exercise 3, Throwing a rock upwards off the edge of a cliff.

I use a kinematics lens because we have motion: an explicit $f(t)$



$$Y(t) = Y_0 + V_0 t + \tfrac{1}{2} a t^2$$

$$0 = \underbrace{100m}_{c} + \underbrace{30\,\tfrac{m}{s}}_{b}\, t \underbrace{- 5\,\tfrac{m}{s^2}}_{a}\, t^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-30\,\frac{m}{s} \pm \sqrt{(30\,\frac{m}{s})^2 - 4(-5\,\frac{m}{s^2})(100m)}}{2(-5\,\frac{m}{s^2})}$$

multiply through by (-1)

$$= \frac{30\,\frac{m}{s} \mp \sqrt{900\,\frac{m^2}{s^2} + 2000\,\frac{m^2}{s^2}}}{10\,\frac{m}{s^2}}$$

$$= 3s \pm \sqrt{29}\ s$$

$$\approx 3s \pm 5.4s = -2.4s,\ 8.4s$$

The negative value.. given this trajectory, if we went backwards in time, it would be at the bottom of the cliff, moving upwards at about $54\,\frac{m}{s}$ ... $= 30\,\frac{m}{s} + g(2.4s)$

But we didn't need the quadradic equation. We knew all along how to find time:

$\Delta X = V_{ave} \Delta t \qquad \Delta t = \dfrac{\Delta X}{V_{ave}} \qquad V_{ave} = \dfrac{(V_0 + V_f)}{2}$

we can use this given constant acceleration (g)

We can find V_f using an energy lens because

$E_p \Rightarrow E_K$ $\qquad E_o = E_f$

$$E_K + E_p = E_K$$

$$mgh_o + \frac{1}{2}mv_o^2 = \frac{1}{2}mv_f^2$$

$$V_f = (v_o^2 + 2gh_o)^{\frac{1}{2}}$$

$$= \left[(30\,m/s)^2 + 2(10\,m/s^2)100\,m\right]^{\frac{1}{2}}$$

$$\approx 54\,m/s \qquad + \uparrow$$

$$V_{ave} = \frac{(30\,m/s + -54\,m/s)}{2} \approx -12\,m/s \qquad \Delta x = -100\,m$$

$$\Delta t = \frac{\Delta x}{V_{ave}} = \frac{-100\,m}{-12\,m/s} \approx \underline{\underline{8.3\,s}}$$

✓

6. 7.6 Exercise 4, Catching the Bus.

PS #6

#1 - Catching Bus

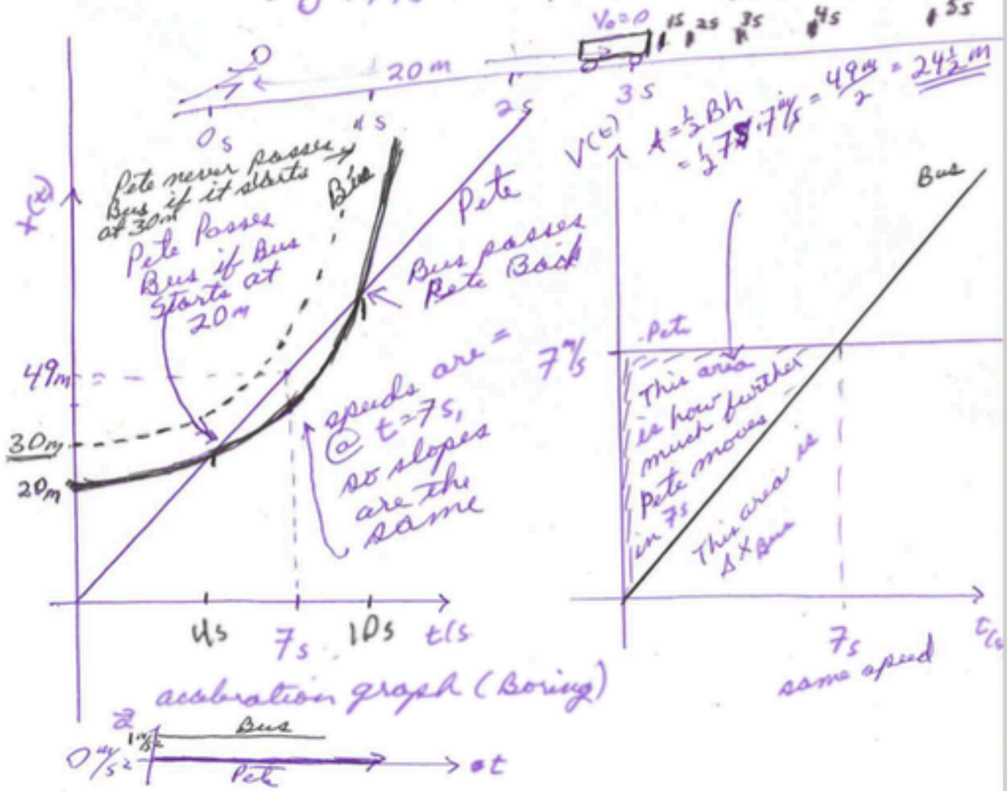
Pete: $V_p = 7\,m/s = \text{const}$, $x_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,m/s^2$, $x_0 = 20m$, $V = a_B t$

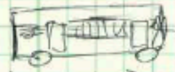
Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $X_p(t) \stackrel{?}{=} X_B(t)$ when and if are our displacements the same

Pete: $X = X_0 + Vt$
$= 0 + 7\,m/s\,t$

Bus: $X = X_0 + V_0 t + \frac{1}{2} a t^2$
$= 20m + 0 + \frac{1}{2}\,m/s^2\,t^2$



6 7.6 Ex. 4

$\vec{V} = 7 m/s$ 

20m $\vec{a} = 1 m/s^2$

Lens: kinematics, we have velocity and $\vec{a}$ as a fn of time.

~~$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$~~

~~$20m + 7 m/s\, t + \frac{1}{2}$~~

Person

$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$

$x(t) = 0 + 7 m/s\, t + 0$

$x_p(t) = 7 m/s\, t$

Bus

$x_B(t) = 20m + 0 + \frac{1}{2}(1 m/s^2) t^2$

$x_B(t) = 20m + \frac{1}{2} m/s^2\, t^2$

$$7 m/s\, t = 20m + \frac{1}{2} m/s^2\, t^2$$

$$0 = \frac{1}{2} m/s^2\, t^2 - 7 m/s\, t + 20m$$

$$t = \frac{7 \pm \sqrt{49 - 4(\frac{1}{2})(20)}}{1} = 7 \pm \sqrt{9}$$

$t = 7 \pm 3$

$t = 4, 10$

$x_p(t) = 7 m/s\, t$

$x_p(4) = 7 m/s\,(4s) = \boxed{28 m}$ @ 4s

$x_p(10) = 7 m/s\,(10s) = \boxed{70 m}$ @ 10s

You catch the bus

7. 7.6 Exercises 5 – 7 (Pulling sled, Hitting a baseball, Torque on a wheel.

7.6 Exercise 5. We would solve this problem exactly as we did before we used trigonometry. The only difference is now we could calculate the components rather than just eyeball (estimate) them. Of course, we recognize this as a dynamics problem whereby the acceleration is horizontal, so we choose x-y components and break the tension into horizontal and vertical components.

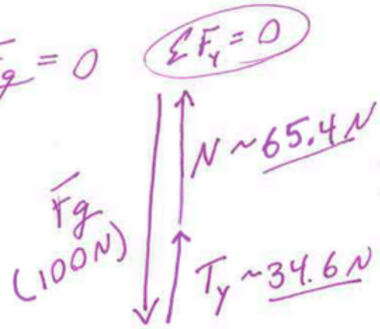
T_x = Tcos(30^o) ~ 40 N(0.866) = 34.6 N*

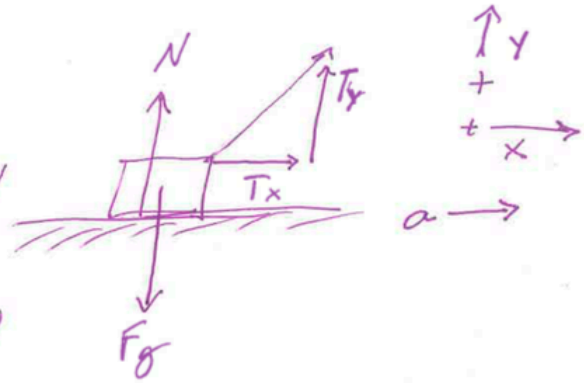
T_x = Tsin(30^o) ~ 40 N(0.5) = 20 N.*

$W = \vec{F} \cdot \overrightarrow{dx}$, We take the x-component of the tension (force) to find that the work I do is *20 N * 5 m = 100 J.*

$$\Sigma F_y = 0$$

$$T_y + N - F_g = 0$$





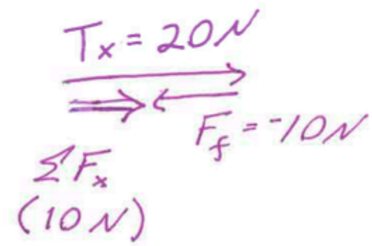
$$\Sigma F_x = ma$$

$$T_x - F_f = ma$$

$$20N - 10N = ma$$

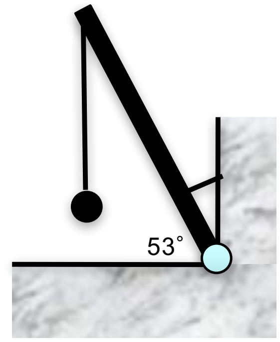
$$\frac{10N}{m} = a \approx 1\ m/s^2$$

$$F_f = \mu N = 0.15 \cdot 65.4N \sim 10N$$



finding the acceleration requires us to use a dynamics lens because the force cause the acceleration. We do a good FBD as always and identify that the forces in the x direction are the horizontal tension and the friction force. To find the force of friction, we need the normal force. We recognize that we are in equilibrium in the y direction because we are (likely) not accelerating off the surface of the earth. Gravity provides 100 N of force (downward), and the vertical component of tension is 34.6 N upward. In order to be in equilibrium in the y direction, the normal force must be 65.4 N (upward). This yields a friction force of about 10 N in the direction opposite to our motion. Assuming that we are moving forward as I pull the sled, the net force is the sum of the x-component of tension minus the frictional force *20 N – 10 N = 10 N* in the positive direction. This yields an acceleration of the 10 kg sled and girl of 1 m/s^2.

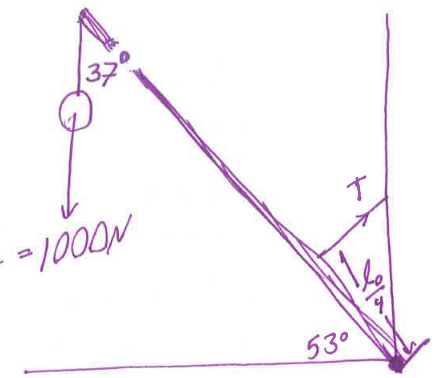
8. In the diagram at right, a post of some length supports a 100 kg ball. The length of the tilted rod is 10 m and the cable is attached 2.5 m from the pivot. From the drawing at right (make your own better drawing), estimate the tension on the cable and the force provided by the foundation at the pivot.



I assign directions

$\longrightarrow +x \quad \uparrow +y$

$\otimes$ + rotation



$F_g = 1000N$

There are unknown forces at the pivot so I use the pivot as the center of rotation, leaving the only unknown torque that of the Tension:

$$\sum \tau_{Pivot} = +T \cdot \frac{l_o}{4} \sin 90° - F_g \cdot l_o \sin 37° = 0$$

(length of rod)

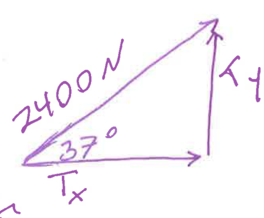
l_o cancels and $\sin 37° \sim \frac{3}{5} = 0.6$

$$T \sim 4 \cdot F_g \cdot \sin 37° \sim 4 \cdot 1000N \cdot 0.6 = \underline{2400N}$$

In order to find the reaction force provided by the pivot, we set $\sum F = ma = 0$. I choose to decompose the Tension into T_x and T_y

$T_y = 2400N \sin 37° \sim 1440N$

$T_x = 2400N \cos 37° \sim 1920N$



$\sum F_x = F_{px} + T_x = 0; \; F_{px} = -1920N\hat{x}$

$\sum F_y = F_{py} + T_y + F_g = 0; \; F_{py} = -440N\hat{y}$

We see that the reaction force that the pivot provides is downward and to the left.

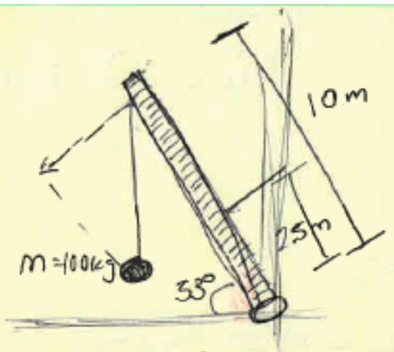
Please show yourself that if the cable was connected in the *middle* of the supporting rod, the tension on the cable would be only 1200 N, resulting in a pivot force that would be 280 N upward and 960 N in the negative x direction.

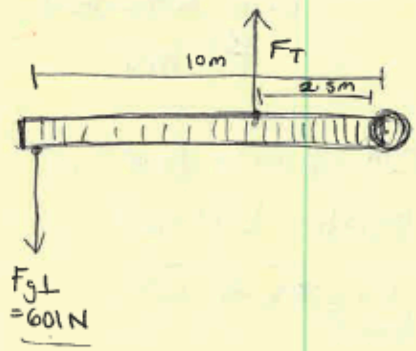
8

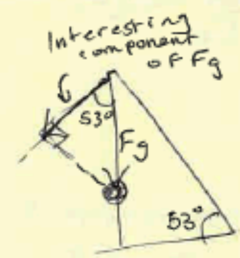
Rotational Dynamics : STATICS

$\Sigma F = 0 \quad \Sigma \tau = 0$

Let's choose



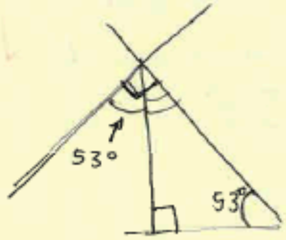




$\cos 53° = \frac{F_{g\perp}}{F_g}$

$0.601\,(1000\,N)$

$= 601\,N$



~~$\tau_g = 6010\,N\cdot m$~~

~~$\tau_T = 601N(2.5m)$~~

$\Sigma \vec{\tau} = (601\,N)(10m) + F_T(2.5m)$

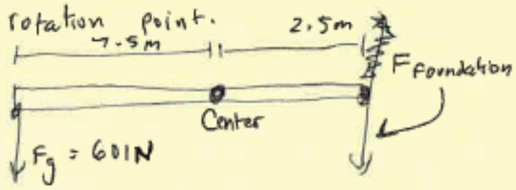
$0 = 6010\,N\cdot m + (-F_T)(2.5m)$

$\frac{6010\,N\cdot m}{2.5m} = F_T$

$\boxed{F_T = 2404\,N}$

TENSION

For finding foundation, change rotation point.



$\Sigma \vec{\tau} = \overset{F_g(r)}{601N(7.5m)} + \overset{F_{foundation}(r)}{(+F_{foundation})(2.5m)}$

$0 = \frac{4507.5\,N\cdot m}{2.5} = F_{foundation}$

$\boxed{F_{foundation} = 1803N}$

Great!

9) In class I asked you to find the coefficient of friction between my computer and a wooden surface. The 114 cm wooden platform was lifted so that one end was 52 cm off the ground when the computer slipped.

a) If the plank made a triangle, find the length of the horizontal component of the incline.

b) Estimate the force of friction and the normal force in terms of mg (the force of gravity on the computer).

c) Calculate the coefficient of friction

d) just for fun, use trigonometry to find the angles of the triangle.

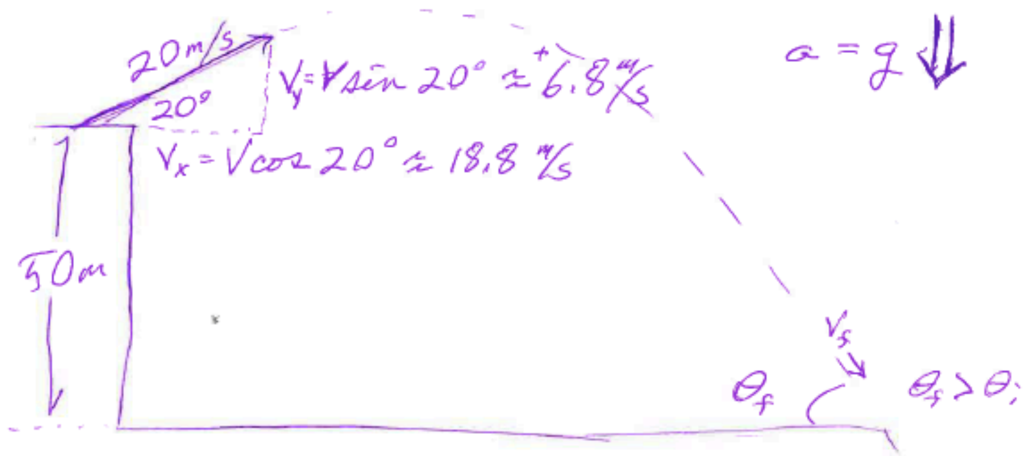
For this solution, please see exercise 4 in section 7.2, the inclined plane. Notice the free body diagram in the left, next to the problem. The key physics is as follows:

- We separate the forces into components sloping down the incline and perpendicular to the incline because there is no acceleration perpendicular to the surface. Thus, the normal force, $N = F_{g\perp}$. Now look at the parallel forces. We raised the incline until the computer started moving… thus, at this point, the maximum frictional force equals the parallel component of gravitational force. $F_f = F_{g\parallel}$. We can't find either of these forces because we don't know the mass, but it doesn't matter because mass cancels: both the normal force and the parallel component of gravity are proportional to mass! Now we use some trigonometry (please do this!) to find:
- $F_{g\parallel} = F_g sin\theta$, and $F_{g\perp} = F_g cos\theta$, Now we can solve for the coefficient of friction, μ, using the equation we know for friction. $F_f = \mu N$: $\mu = \frac{F_f}{N} = tan\theta = \frac{O}{A}$. We can solve the problem either by finding the angle, theta, or by using Pythagorean's Theorem to find the adjacent side of the triangle made by the incline. Please do this both ways, and you will find (using your calculator to take sine inverse of 52 cm / 114 cm) that theta ~ 27 degrees and $\mu \sim 0.51$.
-

10) Hit a baseball off a cliff: Exercise 6, section 7.6

There's two ways to solve this that I know if. Strictly kinematics, you can make a good drawing and decompose the initial velocity into vertical and horizontal components. We do this because of dynamics because (Gravitational) force cause acceleration (downward). It is the *time* that connects the vertical situation (the ball goes upward, stops, comes downward, with downward acceleration of gravity) while the in the horizontal direction, the ball moves forward at a constant horizontal speed until it hits the ground. It is TIME that connects the two – the ball only moves horizontal for the same amount of time that it is moving up and down. We solve the vertical (quadratic) equation for time, and substitute it into the horizontal equation for constant speed in the x direction to get the distance the ball goes forward before hitting the ground. Then we can look at the vertical velocity! We use $v_f = v_i + -gt$ to find the final vertical velocity and add this to the horizontal velocity in order to get the final velocity. We use trig to find the angle.

But, I like energy! First I'd make a good drawing. I would use energy to solve this problem because $E_k + E_g \Rightarrow E_k$. Using this, I find v_f then v_{yf} the time, then angle, then distance.



I'll use an energy lens because

$E_K + E_g \Rightarrow E_K$

$\frac{1}{2}\cancel{m}V_i^2 + \cancel{m}gh_i = \frac{1}{2}\cancel{m}V_f^2 + \cancel{mgh_f}^{\to 0}$

$V_f^2 = V_i^2 + 2gh = (20\,m/s)^2 + 2(10\,m/s^2)50\,m$

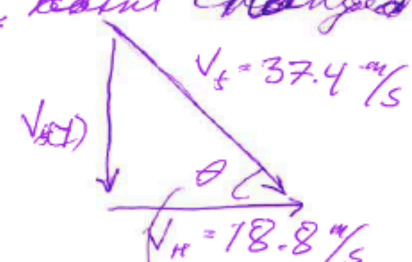
$= 400\,m^2/s^2 + 1000\,m^2/s^2$

$= 1400\,m^2/s^2$

$V_f = (1400\,m^2/s^2)^{\frac{1}{2}} = 37.4\,m/s$

reconstructing V_f, we know V_x hasn't changed

$V_H = V_x = 20\,m/s \cos 20° \approx 18.8\,m/s$



using Pythagorus

$\cancel{V_{fT}}$ $V_{fy}^2 + V_H^2 = V_f^2$

or $V_{fy} = \cancel{-41.9\,m/s} \to 32.4\,m/s = V_{iy} + -gt$,

$\theta = \tan^{-1}\left(\frac{32.4\,m/s}{18.8\,m/s}\right) = 59.9°$

$a = \frac{\Delta V}{\Delta t}$, so $\Delta t = \frac{\Delta V}{a} = \frac{V_f - V_i}{a} = \frac{-\cancel{41.9}\,32.4\,m/s - 6.8\,m/s}{-10\,m/s^2}$

$= \sim 3.9\,s$

Then we can find the final distance because we know the horizontal speed and the time:

$$\Delta x = v_x * \Delta t = 18.8\frac{m}{s} * 3.9s = 73\,m$$

Now that we've done this using energy, we can do the straight kinematics solutions using the displacement and velocity functions of time… "explicit functions of time".
Using the same drawing above for energy, we separate the motion into x and y components:

x, horizontal $a=0$

$x_f = \cancel{x_i} + V_i t + \cancel{\frac{1}{2}at^2}$, $V_i = 18.8\,m/s$

$x_f = 18.8\,m/s \cdot 3.9\,s = 73.6\,m$

$\cancel{= V_i}\Delta t$

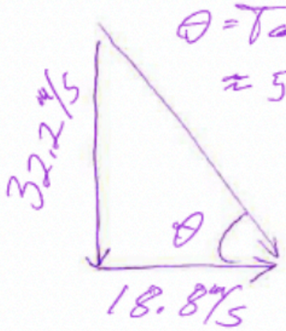
$V_f = V_i + \cancel{at} = V_f = 18.8\,m/s$

$V_f = \left((32.2\,m/s)^2 + (18.8\,m/s)^2\right)^{\frac{1}{2}}$

$= 37.1\,m/s$

$\theta = \tan^{-1}\left(\frac{32.2}{18.8}\right)$

$\approx 59.7^\circ$



y, vertical $a = -g$

$y_f = y_i + v_i t + -\frac{1}{2}gt^2$, $V_i = 6.8\,m/s$

$0 = \underbrace{y_i - y_f}_{50\,m = c} + \underbrace{v_i}_{b} t - \underbrace{\frac{1}{2}g}_{a = -5\,m/s^2} t^2$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-6.8\,m/s \pm \left((6.8\,m/s)^2 - 4(-5\,m/s^2)(50\,m)\right)^{\frac{1}{2}}}{2(-5\,m/s^2)}$

$= .68\,s \pm 3.24\,s$, $-2.6\,s$, $3.9\,s$

3.9 s

$V_f = V_i + at = 6.8\,m/s - 10\,m/s^2(3.9\,s)$

$= 6.8\,m/s - 39\,m/s$

$\approx -32.2\,m/s$