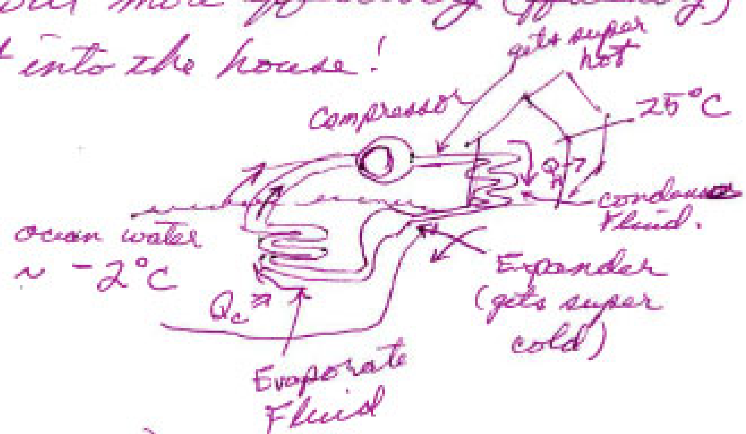
Schwartz Problem Set #7

1) Say you live in Alaska where it's routinely -40 °C for days on end. You want to keep your house at 25 °C and you presently burn natural gas with an efficiency of about 85% (some heat remains in the exhaust in order to not condense the water vapor). However, you're considering using (NGCC) electricity to heat your house with a heat pump. You're happy that there's a narrow but deep ocean inlet next to your house that never freezes all winter long!
   a) Why are you happy to have this ocean inlet?
   b) Draw a picture describing how this heat pump would work. Include all mechanisms and the house and area as well.
   c) Estimate the ideal coefficient of performance for heating your house using the ocean inlet.
   d) In order to drive the heat to the evaporator coils from the water, you need 10 °C difference and you need 15 °C difference between the condenser coils and the room to drive heat into the room. NOW what is the best coefficient of performance you would get?

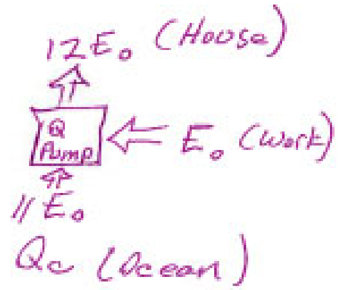
1) a) $COP_{max} = \frac{T_H}{T_H - T_C}$ By having a warmer place to go Extract heat from (0°C > -40°C), the heat pump will more effectively (efficently) pump heat into the house!

b)



c) $COP \leq \frac{(25°C + 273K)}{(25°C - -2°C)}$

$\lesssim \frac{300K}{25K} \sim \underline{\underline{12}}$



D) $COP \leq \frac{(35°C + 273K)}{\Delta T}$

$\approx \frac{310K}{50K} \approx \underline{\underline{6}}$

$T_H \Rightarrow 35°C$

$T_C \Rightarrow -12°C$

$\Delta T \sim 50°C$

2) You don't ever get your ideal COP for a heat pump for the same reason that we don't have heat engines achieving the Carnot Thermodynamic Limit. We practically can't perfectly operate (for instance) a perfect isothermic process if we're in a hurry to get things done. A reasonable COP for such a cold place might be about 4. Please compare the following three technologies:
   a) Burning natural gas whereby you extract about 85% of the chemical potential energy from the natural gas.
   b) Using an electric heater in your house. Electric heaters are all 100% efficient (despite what advertisements might say along the lines of "high efficiency"). All the electricity turns to thermal energy.
   c) Using an electric heat pump.

For the above three scenarios, please compare how much you would spend to heat your house, and how much CO_2 would be emitted. Let's see… you need something to compare it to? Using a space heater, you would use 1000 kWh per month. In Alaska, they pay about \$0.16/kWh just like us. So, given these two numbers, please calculate the costs and CO_2 emissions (kg of CO_2/month) for all three scenarios.

2) ok $Q_H \sim 4 W_{(Electric)}$

Electricity $\Rightarrow \frac{1}{3}$ kg(CO2)/kWh , \$0.16/kWhr

NG: ~ \$0.01/CF , ~ \$10.00/Thousand (Ft³)

Wikipedia claims 10^3 ft³ ≈ 10^6 BTU, or GJ
↑ ~10^3 J

Let's just say we use 10^3 kWh/month in Electrical energy for a (100% efficient) space heater:

b) Space heater $W_{in} = Q_H = 10^3$ kWh

cost: \$0.16/kWh · 10^3 kWh = \$160 ouch!

$CO_2 = \frac{1}{3}$ kg CO_2/kWh · 10^3 kWh = 333 kg(CO_2) guilt!

a) Natural gas: $E_{(methane)} \times$ efficiency $= Q_H = 10^3$ kWh

$$E_{(methane)} = \frac{10^3 \text{kWh}}{.85} \approx 1150 \text{kWh} \approx 1200 \text{kWh}$$

$$= 1.2 \times 1000 \times 3.6 \times 10^6 \text{J} \approx 5 \times 10^9 = 5 \text{GJ}$$

cost:
5 GJ · \$10.00/GJ = \$50.00

CO_2 Natural gas

$$5 \text{GJ} \cdot \frac{15 \times 10^3 \text{g(C)}}{\text{GJ}} \cdot \frac{44 \text{g}(CO_2)}{12 \text{g(C)}} \approx 5 \cdot 15 \cdot \frac{44}{12} \text{kg} \approx 250 \text{kg } CO_2$$

3) You build a wind turbine for Dong! The next generation. Remember the last slide in Friday's lecture?: So take a look at the present size… yes, the radius is close to a football field in length. OK, now double it. And out in the North Sea, you expect to gather electricity at winds all the way up to 15 m/s!
   a) Calculate the amount of power one of these turbines will put out in 15 m/s wind, if we are able to achieve the Betz Limit.
   b) You can't really achieve the Betz Limit. How close will you come? Are we "pretty close" to the Betz Limit?

c) Heat Pump: COP = 4 $\qquad Q_H = W \cdot COP$

$Q_H = 10^3$ kWh $\qquad W_{in} = \frac{10^3 \text{ kWh}}{4} = 250$ kWh

Cost = 250 ~~kWh~~ · \$0.16/~~kWh~~ = \$40.00

$CO_2 = \frac{1}{3}\frac{kg\ CO_2}{kWh} \cdot 250\ kWh \approx 83\ kg\ CO_2$

Yeah, the heat pump wins! Well, maybe not... It would have to cost thousands to install ~~such~~ such an elaborate system, compared to a very inexpensive space heater or N.G. ~~Heat~~ Heater.

3) Max Power! $\quad V_{max} = 15\ m/s$

$\rho_{air} \approx 1.3\ kg/m^3$

$A = \pi r^2 = \pi (524\ m)^2$ ← $\sim 5 \times 10^2$

$\approx 8 \times 10^5\ m^2$

$P_{max} \approx \frac{1}{2}(1.3\ kg/m^3)(15\ m/s)^3 \cdot 8 \times 10^5\ m^2$

$\approx$ 2 GW... Impossible!

times Betz limit (0.59) ⇒ 1 GW... still I don't believe it. We can achieve ~45% efficiency

I KNOW the radius was in feet $r \Rightarrow \frac{r_0}{3}$ $A \Rightarrow \frac{A_0}{10}$

so $P_{Turbine} \approx$ 100 MW still Pretty good!