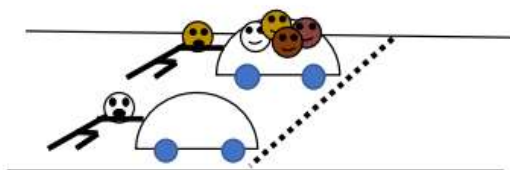


**CAREFUL!** What conservation laws are, and what they're NOT.

Conservation means that something doesn't change in *time*, it doesn't mean that it's the same as something else. For instance, in conserving mass, we know that  $m_i = m_f$ , but this doesn't mean that  $m_A = m_B$  for two different objects. Students mix up these two relationships! Please let me illustrate with a ridiculous example and a more challenging subtle example.

**Ridiculous:** A truck drives off a cliff and lands next to a little girl as she jumps down from a step. Please compare the mass, momentum, and kinetic energy of the girl to the truck just as they hit the ground. Would you say  $m_G = m_T$ ,  $p_G = p_T$ , and  $E_G = E_T$ , because mass, momentum, and energy are always conserved? Certainly not. These are totally different bodies doing different things. They have different masses, momentum, and energy. What the conservation laws *do* state is that the mass of the truck doesn't change as it falls  $m_i = m_f$ . For energy, the total energy of the truck after the fall is the same as before, but it changes forms. So, the gravitational potential energy lost = kinetic energy gained. Lastly, we might be compelled to conclude that momentum is conserved, but there is an outside force on the truck, gravity. Thus, there is a large change in momentum =  $F_g * \Delta h$ .

**More subtle example:** Starting from rest, two identical cars with low friction wheels are pushed by two friends with the same force on a smooth surface. After 10 seconds please compare the momentum and energy of the two cars.



**Momentum Lens:** You might be tempted to say, “the momenta are equal because momentum is always conserved!” But you can see that these are different bodies, so such a statement makes no sense. So, we ask ourselves, “what’s happening with momentum?” There’s an outside force, so the momentum *isn't* conserved. The *change in momentum*, the impulse each car receives is:

$$\Delta p = F * \Delta t.$$

However, in this case, the force and time for the pushing is the same for each car. So, they do gain the same amount of momentum in 10 seconds.

**Energy Lens:** You might be tempted to say, “the energies are equal because energy is always conserved!” But again, we realize that energy conservation compares energy over time, not between two different bodies. So, with an energy lens, we consider the energy transition:

$$W_{external} = F * \Delta x \Rightarrow E_{kinetic}.$$

We know each car has the same force, but we have to look through a dynamics lens to compare the distances they are pushed, because the forces cause the cars to accelerate. The empty car with the smaller mass will have the larger acceleration. Using kinematics, we can see that the empty car has a higher final speed, and therefore a higher average speed and therefore a larger total distance traveled. Thus, the empty car will have more kinetic energy after 10 seconds.