Chapter 4: Introduction to Rotation

<u>We started</u> this course with four lenses of mechanics.

- Momentum: $\vec{p} = m\vec{v}$; system momentum is always conserved; momentum is a vector.
- Energy: Energy is a scalar, is always conserved, but can change forms.
- Dynamics: $\sum \vec{F} = m\vec{a}$, we have a protocol to solving these. We practice it.
- Kinematics: Explicit time dependence of motion: position, velocity, acceleration.

We start now with the rotational analogues of these four lenses:

- <u>Angular Momentum</u> $\vec{l} = I\vec{\omega}$; is how hard it is to stop a body from spinning. Just like *linear momentum,* angular momentum is a vector, and in an isolated system is conserved. The initial example we had for linear momentum conservation was two carts colliding. The rotational analogue would be if you dropped a stationary mass onto a rotating platform. In an inelastic collision, the would stick together and rotate, but at a lower rotation rate than the platform was previously spinning.

- <u>Rotational Kinetic Energy</u>: We introduce a new form of kinetic energy $E_R = \frac{1}{2}I\omega^2$. If something is spinning, then the different parts have different speeds constituting kinetic energy.

- <u>Rotational Dynamics</u>: Turning forces, or "torques" cause rotation rates to change (rotational acceleration): $\sum \vec{\tau} = I\vec{\alpha}$. rotational inertia (or moment of inertia) *I*, is the resistance to *angular* acceleration. Large, massive objects have a larger "*I*" and are harder to rotationally accelerate (start and stop spinning). Just like with linear forces, a torque is a single turning interaction between two bodies, affecting each in opposite directions. So, if I put a torque on an object, it puts an opposite torque on me.

- <u>Rotational Kinematics</u>: Explicit time dependence of rotational motion:
  - angle (θ),
  - rotational velocity ($\omega = \frac{d\theta}{dt}$) or how fast something is spinning, and
  - angular acceleration ($\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$) or how fast this rotation is speeding up or slowing down.

  These quantities are also vectors, just like the linear analogues because you can rotate an object about any axis in either direction. For instance, if two objects are spinning in opposite directions at the same ω, the angular velocities of each object would have opposite signs, as would their angular momenta.

Example 1:
Two identical disks ("A" and "B") are spinning in opposite directions in space, and $\omega_A = \underline{3}\omega_B$.
How would you compare:

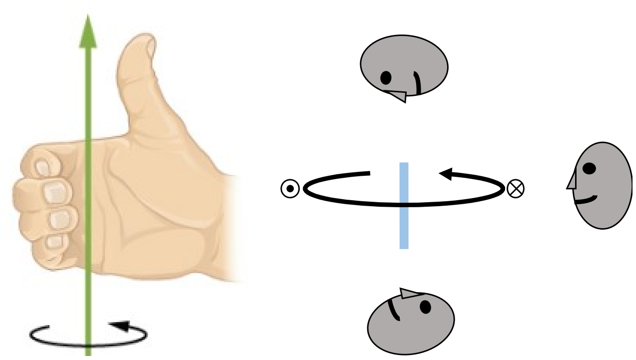
- Their initial angular momentum? $l_A = \_\_l_B$
- The two initial kinetic energies? $E_{RA} = \_\_E_{RB}$

The two bodies collide and stick together:

- What is the linear analogue for this situation?
- What lens is most important for use here?
- How does the final rotational velocity compare to the initial rotational velocity of A, ω_A?
- Is kinetic energy lost in this collision? If not how do you know? If it is, please find an expression indicating how much energy is lost.

Example 2:
A low friction cart and a large rolling sphere are moving with equal speed on flat ground headed toward a gradual uphill. They continue up the hill and then return back. Do they both go the same distance up the hill, or does one of them go further before stopping and turning around? Can you explain why? Which lens did you use to explain your reasoning?

4.1 Rotation Direction



Suppose there is a bicycle wheel lying flat on the ground rotating as shown by the arrow in the diagram at right.

- The rim to the right is moving away from you (the reader). We indicate "away from you" by a circle with an "x" in it, like you are see the tail feathers of an arrow flying *away* from you.
- The left section of the spinning wheel is coming out at you. "Into your face" is indicated by concentric circles as if you are looking at the point of an arrow coming straight at you.

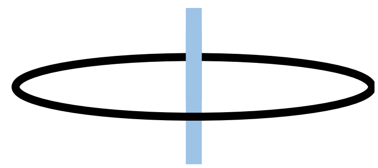
We define the direction of rotation with the "Right-Hand Rule" (RHR): Curl your fingers in the direction of the rotation *with your right hand,* and your perpendicular thumb will point in the direction of the rotation! This seems easy, but most people need to practice and have discussions with others.

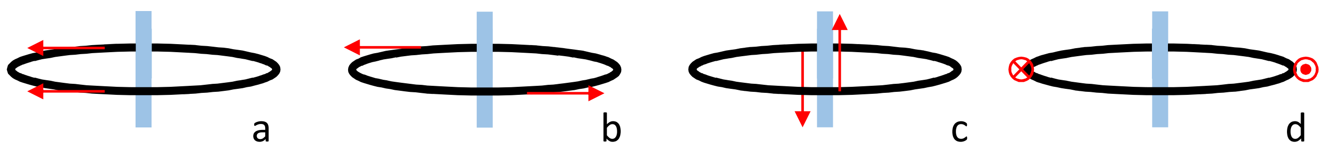
"Why can't we just use 'clockwise' and 'counter clockwise'?" – Our "clockwise" convention of direction only communicates direction to people standing where you are. Please see the diagram (above right) of the three people looking at the spinning wheel. The person on the bottom says it's turning clockwise while the person above claims counter-clockwise. The person to the right would say "the left side is coming toward me and the right side is moving away." None of these people would be able to agree. However, each one of them could orient their right hands consistent with the drawing and agree that the rotation is in the upward direction.

"How could physics possibly depend on using your *right* hand?" – It doesn't. Using your right hand is totally arbitrary. You could use your left hand (if you used it all the time) or always draw a picture to define direction and you would still get the same answers. We arbitrarily pick this right hand convention so that the world can communicate direction.

<u>Example 1</u>

At right, there is a wheel lying in the horizontal plane with a vertical axis in light blue. I want to rotate it so that $\vec{\omega}$ is pointing to the left, or the $-\hat{x}$ direction. Which diagram below indicates how I would orient two forces (in red) in order to make the wheel rotate in the $-\hat{x}$ direction?



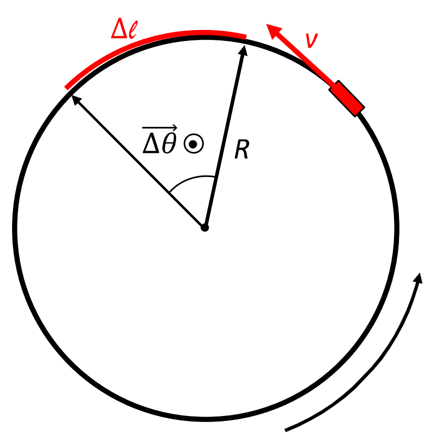


Answer on next page, don't look yet!

a) This set of forces would result in *linear* acceleration in the $-\hat{x}$ direction, but no angular acceleration.
b) This set of forces would result in rotation in the upward direction.
c) This set of forces would result in no net force, but a torque to the left, and rotational acceleration in the $-\hat{x}$ direction!
d) This set of forces would result in torque and rotation in the downward direction.

4.2 Connection Between Rotational and Linear Motion

Imagine you are looking downward upon the bicycle wheel from last chapter and you rotate it some angle $\overrightarrow{\Delta\theta}$ in the direction pointing straight out at you as shown at right. You do this in some period of time, so you can calculate $\vec{\omega}$, pointing in the same direction. Then you slow down the rotation, so there is angular acceleration, $\vec{\alpha}$, pointing away from you. As you do this, you must see the rim of the wheel moving, and know that each point on the rim has moved some tangential distance and has some tangential speed and tangential acceleration because of the rotational motion.



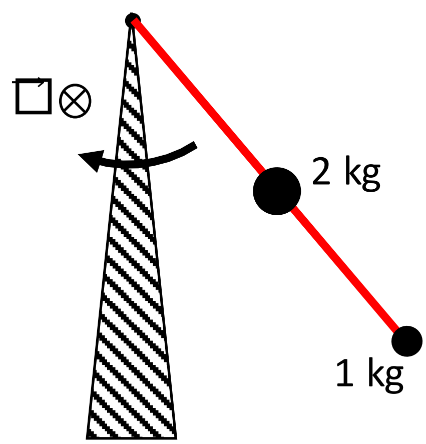
From the definition of angle in radians:
$\theta = \frac{l}{R}$, or $l = R(\theta)$, where l is the length that the point on the rim has moved and R is the radius of the wheel. Radians is unitless because it is a ratio of how many radius lengths are in the arc of the angle. One radian defines an arc of one radius. We can see that a radian is a little less than 60° since an equilateral triangle has three 60° angles. If one of the sides were bent into the arc making the triangle a sector of a circle, it would decrease the opposite angle a little as shown above by the sector defined by angle $\overrightarrow{\Delta\theta}$.

How do we find the tangential speed and tangential acceleration in terms of angular motion? We find the time rate of change of our equation for θ above noting that R is constant:
$v_{tan} = \frac{d}{dt}(l) = \frac{d}{dt}(R\theta) = R\frac{d\theta}{dt} = R\omega$, and if we take the time rate of change again, we see $a_{tan} = \frac{d}{dt}(v_{tan}) = \frac{d}{dt}(R\omega) = R\frac{d\omega}{dt} = R\alpha$.

We see that the tangential distance, speed, and acceleration are found by multiplying R by the rotational analogues: θ, ω, α.

<u>Example 1</u>:
At right, a red rigid rod, driven by a motor, rotates about a pivot at the top of a triangular support at $\vec{\omega} = \frac{10}{s}$, or 10 radians per second. The 2 m long rod is very light, but has a 1 kg mass at the end and a 2 kg mass in the middle, half way between the pivot and 1 kg mass.

a) Without a calculator, please estimate this rotation rate. Is it about 100 rpm?
b) Find the speed of each mass.
c) Find the kinetic energy of each mass.
d) What is the rotational kinetic energy of the rotating structure (the sum of kinetic energies)?

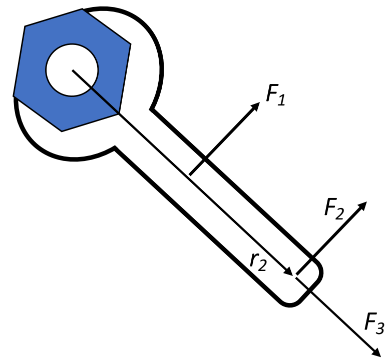


When a wheel is rolling without slipping (using static friction), the speed of the portion of the rim touching the ground is always zero. Consequently, what we might have calculated for the motion of the rim, is actually the motion of the wheel hub, or of the vehicle.
<u>Example 2</u>:
With a drawing of a bicycle, please acquaint yourself with the following scenario. You are riding a bike with 700 mm wheels (diameter = 0.700 m). Starting from rest, you smoothly accelerate in 5 s, to a speed of 10 m/s toward the right =>.

a) Is this possible? Could you do this on level ground?
b) Calculate $\vec{\omega}_f$, including direction.
c) Calculate $\vec{\alpha}$, assuming it is constant.
d) Calculate $\overrightarrow{\Delta\theta}$, the angle that the wheel has rotated through.
e) What is the velocity of the rim at the top of the wheel?

4.3 Torque and Rotational Work and Power

Torque, $\vec{\tau}$, is "rotational force":
$\tau = F_{\perp}r$, where $F_{\perp}$ is the force perpendicular to the radius, r, also called the "moment arm" or "lever arm", and the direction is determined by RHR, section 4.1. If you want to get a box to slide, you apply a force that is strong enough to overcome friction. If you want to turn a nut that is stuck you need *torque*. F_3 at right won't turn the nut because a force in the radial direction produces no torque. If all the forces are the same magnitude (strength), then F_2 will produce greater torque than F_1, because F_2 is further from the center of rotation.



Torque has units of Nm. We might think that this is also *Joules*, but in mechanics *Joule* is only for work or energy. Torque alone is not work, because if nothing moves, no work is done and there is no energy conversion. When the nut rotates through some angle, $\Delta\theta$, then your hand producing the force moves a distance $r\Delta\theta$, and work is done. Work is *force x distance* in the same direction for linear systems. For rotational systems, work is *rotational force x rotational distance* (or *torque x angle*), in the same direction:
$W = F\Delta l = \tau\Delta\theta$, in this case let torque and $\Delta\theta$ both point out at you.

$\Delta\theta$ has no units (radian is just a ratio of arc length to radius). So, both work and torque have units of Nm. Once you've multiplied by $\Delta\theta$, there is motion and work has been done, and we are able to change Nm to J.

Exercise 1:
Prove the above relationship by applying F_2 tangentially (as in the diagram) while your hand moves some tangential distance, dl, as the nut rotates through some angle, $\Delta\theta$.

Exercise 2:
Imagine pulling on a wrench against a sticky nut as shown above by F_2. You pull with a force of 200 N at the end of a 40 cm wrench and rotate the wrench one full rotation in 2 seconds.

- Calculate the torque and the corresponding rotational work you do.
- Where did this energy go?
- Calculate your rate of power production during this time.

In exercise 1 in section 2.1 (Power), you show power is the rate at which you do work and is equal to *force x speed* in the same direction.

$$P = \frac{W}{\Delta t} = \frac{F\Delta l}{\Delta t} = Fv$$

Similarly, for rotational motion, $P_{rot} = \tau\omega$, or *rotational* force *x rotational* velocity

Exercise 3:
Please show that $P_{rot} = \tau\omega$

Exercise 4:
Say you are bicycling. You are able to push the pedals with a constant force of about 200 N and the radius of the pedal shaft (crank arm) is 175 mm. Most people can spin their legs at 90 rpm.

- Find the torque and use it to find your power output.
- Find ω, and use it to find the power output.
- Find the tangential speed of the pedal and use it to find your power output. Hopefully, it's the same as you got above.

Exercise 5:
A 2017 Ford Mustang with a 3.7 liter V6 engine provides 300 HP and 280 foot-pounds of torque! Let's unpack this:

- A "V6" means that there are 6 cylinders where exploding air/gas drive a piston downward to rotate the drive train to rotate the wheels. There are two rows of 3 pistons next to each other arranged in a V pattern. Learn more about internal combustion from animations at http://www.animatedengines.com/otto.html
- 3.7 liters is almost a gallon. This is the total volume of all 6 of the piston chambers added together.
- Please convert the power and torque to metric units.
- Find the rotation rate of the engine under the conditions that provide this power and torque at the same time. Please express the rotation rate in both radians/s and RPM.
- Actually, we don't get maximum power and torque at the same rotation rate, ω. Please explain why an engine produces maximum power at a higher rotation rate than it achieves maximum torque.

4.4 Moment of Inertia

The Moment of Inertia, or Rotational Inertia is used for rotational mechanics the same way mass is used in linear kinetic energy and dynamics: mass is a measure of how hard it is to accelerate a body. The moment of inertia is a measure of how hard it is to *rotationally* accelerate a body. From 4.0, we have:

- Angular Momentum: $\vec{l} = I\vec{\omega}$
- Rotational Kinetic Energy: $E_R = \frac{1}{2}I\omega^2$
- Rotational Dynamics: $\sum \vec{\tau} = I\vec{\alpha}$

Exercise 1:
Examine the three quantities above

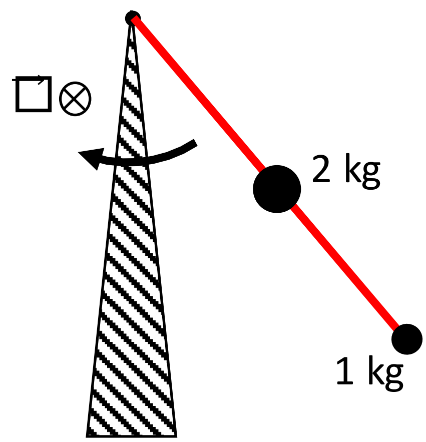
- How do they compare to their linear analogues of momentum, kinetic energy, and dynamics?
- Which one(s) have the same units?

- Please substitute linear quantities in the formulas above to show these formulas are in fact correct.

<u>Why don't we just use mass?</u>
In a rotating body, all the individual masses have different speeds depending on their distance from the rotation center, but all masses have the same *angular* displacement, *angular* speed, and *angular* acceleration θ, ω and α. Thus, we need a different quantity to represent resistance to angular acceleration. We assert three things about the moment of inertia and then show how these assertions are correct:

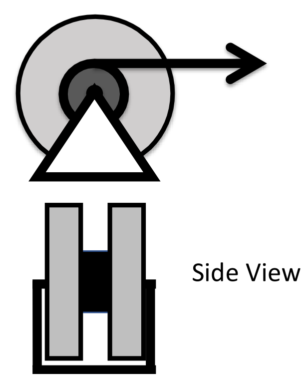
- For a point mass rotating about a pivot some distance *r* away, $I = mr^2$.
- For a rigid structure of many masses, the total moment of inertia is the sum of the moments of each mass: $I_{total} = \sum_i^n I_i = \sum_i^n m_i r_i^2$.
- For a solid body, the above sum becomes an integration over the entire mass. However, integration is done over *spatial coordinates* not mass. So, the integration is converted to a spatial integration and multiplied by density: $I_{body} = \int dI = \int (dm) r^2 = \int \rho (dV) r^2$

<u>Exercise 2</u>:
At right, a red rigid rod rotates about a pivot at the top of a triangular support at $\vec{\omega} = \frac{10}{s}$, or 10 radians per second. The 2 m long rod is very light, but has a 1 kg mass at the end and a 2 kg mass in the middle, half way between the pivot and 1 kg mass.



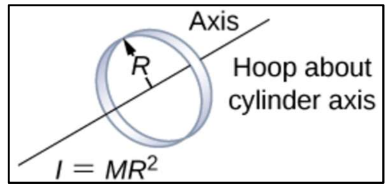
e) Find the moment of inertia of each mass about the pivot point, and the total moment of inertia of the structure.
f) Find the rotational kinetic energy of the spinning structure.
g) Does your answer agree with example #1 in section 4.2?
h) Which of the two methods do you prefer for finding E_R?

<u>Exercise 3:</u>
A yo-yo device is mounted on a stand and I pull on a 2 m string wrapped around the center with a force of 8 N. The wheel starts from rest, has a mass of 2 kg, and center shaft and body radii of 20 cm and 40 cm, respectively.



a) After I pull for 2 seconds, find the wheel's angular velocity.
b) If a bug is on the top of the wheel at that moment (2 seconds), find the bug's acceleration, indicating the correct x and y directions.
c) Find the power I am putting into the yo-yo at t = 2 s.

4.5 Moments of Inertia



In section 4.4, we showed the moment of inertia for a point mass is $I = mR^2$, and that for many masses, you just add up the moment of inertias of all the masses. However, a solid body rotating about an axis has an infinite number of points, so the moment of inertia (even for objects of the same mass and size) will depend on how the mass is distributed in that body. Only an infinitely thin hoop (above, right) has a moment of inertia of mR^2 because all the mass is at the same radius. Solid bodies have a lot of mass at smaller radii, so we'd expect them to have a moment of inertia smaller than mR^2.

Exercise 1:
Consider the following round objects, all of the same mass, M, and radius, R. Consider their mass distribution – how far is the mass from the axis of rotation? Can you put them in order of highest moment of inertia to lowest moment of inertia and justify your reason?

- A hoop as shown above right, spinning about an axis perpendicular to the plane of the hoop.
- A hoop as shown above right, but spinning about a vertical axis *in the plane of the hoop*, as you might spin a "hula-hoop" on the ground to make it "stand up".
- A solid sphere.
- A thin-walled hollow sphere.
- A solid disk spinning about an axis perpendicular to its surface.
- A solid disk spinning about an axis in the plane of its surface, such as spinning a coin on a table top.

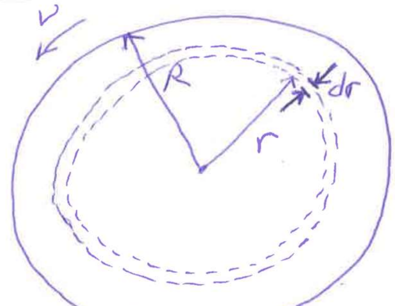
Exercise 2:
From Exercise 1, consider each of three objects of equal mass and radius rolling with equal speed on flat ground headed toward a gradual uphill: A hoop, a disk, and a solid sphere. They all roll up the hill, and back down again. Do they all go the same distance up the hill, or does one of them go further before turning around? Can you explain why? Which lens did you use to explain your reasoning?

In order to find the exact moment of inertia, we integrate each bit of mass:
$I_{body} = \int dI = \int (dm) r^2 = \int \rho (dV) r^2$, which is an excellent calculus exercise.

Example 3:
Consider a disk of radius, R and mass M, spinning about its center as in the image at top, except that it is a *solid* disk.

- Would the moment of inertia be greater than, less than, or equal to MR^2? Provide a reason for your answer. *Because most of the mass is located a distance $r<R$ from the center of rotation, this mass has a smaller radial distribution than a ring (hoop) of radius R, and thus must have a moment of inertia less than MR^2.*
- Please do the integration to find the actual moment of inertia of this disk.

Find moment of inertia of a coin Spinning on a axis coming out at you, $\vec{\omega}\odot$



This is a straight-up math problem, no mechanics lens.

$$I = \int_{mass} dI = \int_{mass} r^2\,dm = \int_A r^2 \sigma\,dA$$

we assume a disk of uniform $\overbrace{\text{surface density}}^{\sigma}$ $\left(\frac{kg}{m^2}\right)$ such that $M = \sigma A = \sigma \pi R^2$, or $\sigma = \frac{M}{\pi R^2}$

we know that $I_{ring} = mr^2$ because all the mass is at the same radius, r. So, we just need to integrate all the rings of thickness dr from $r=0$ to $r=R$ $\quad dm = \sigma\,dA = \sigma \underbrace{2\pi r\,dr}_{\text{area of ring}}$

$$dI = r^2\,dm = r^2 \sigma 2\pi r\,dr$$
$$= 2\pi\sigma r^3\,dr$$

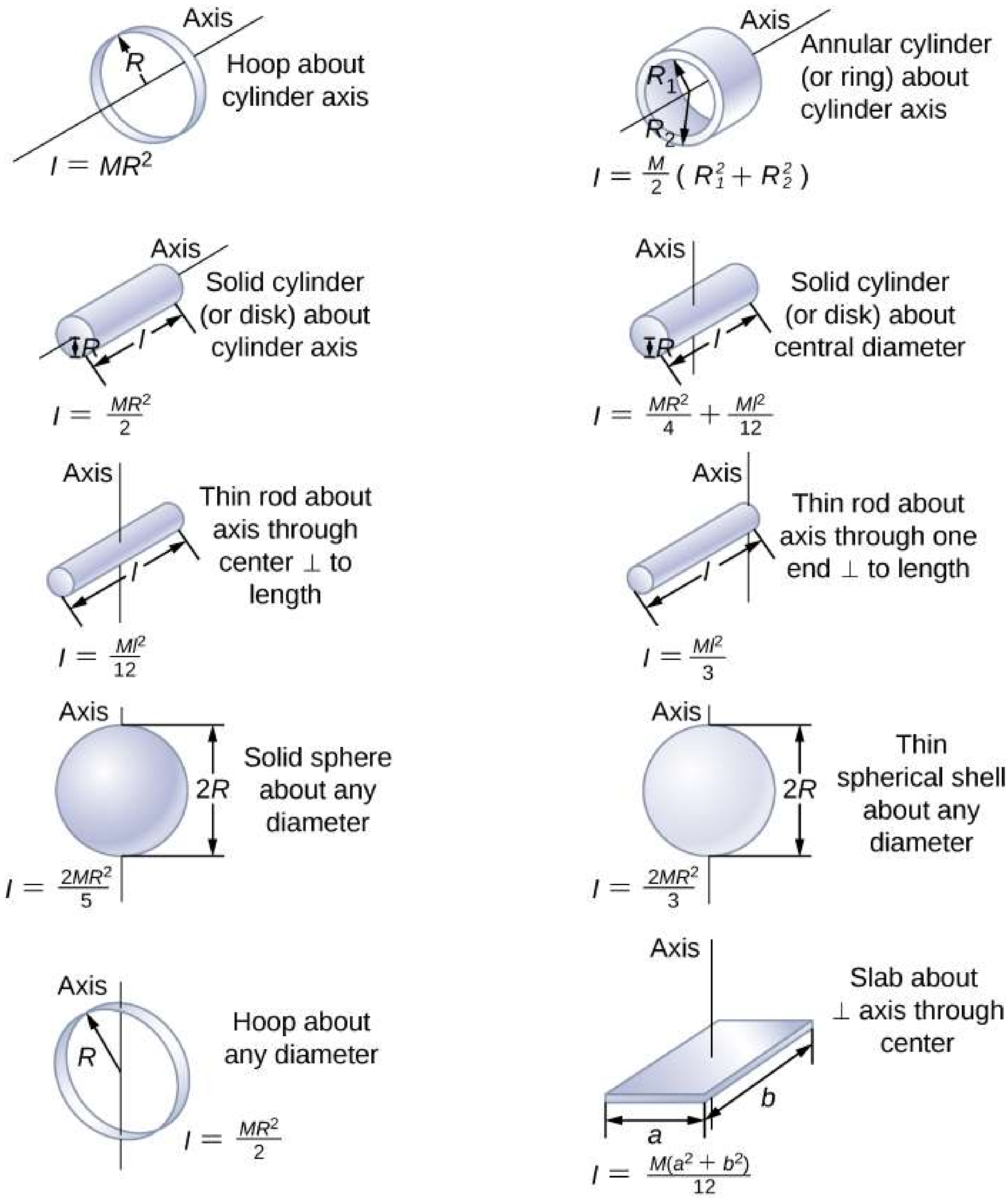
$$I = \int_{r=0}^{r=R} dI = \int_0^R 2\pi\sigma r^3\,dr = \underbrace{2\pi\sigma}_{constant}\int_0^R r^3\,dr = 2\pi\sigma\frac{R^4}{4}$$

substituting in our value for $\sigma = \frac{M}{\pi R^2}$,

$$I = \cancel{2}\cancel{\pi}\frac{M}{\cancel{\pi}\cancel{R^2}}\frac{\cancel{R^4}^{\,2}}{\cancel{4}_{2}} = \frac{1}{2}MR^2,$$ so the moment of inertia for a <u>disk</u> is half that of a ring of the same mass + radius because much of its mass is located at smaller $r < R$.

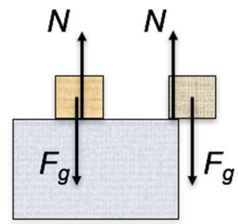
<u>Example 4</u>:

Find the exact moment of inertia for the other objects in Exercise 1. Check your answers by looking up the accepted moment of inertia of each. Were your Exercise 1 predictions correct for the order you put the objects in?

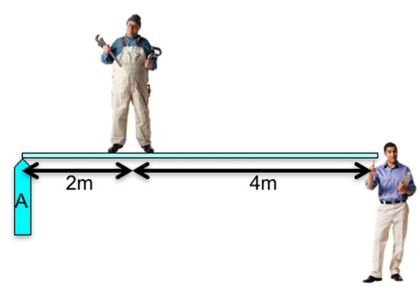

Axis
R
Hoop about cylinder axis
$I = MR^2$
Axis
R_1
R_2
Annular cylinder (or ring) about cylinder axis
$I = \frac{M}{2}(R_1^2 + R_2^2)$
Axis
R
l
Solid cylinder (or disk) about cylinder axis
$I = \frac{MR^2}{2}$
Axis
R
l
Solid cylinder (or disk) about central diameter
$I = \frac{MR^2}{4} + \frac{Ml^2}{12}$
Axis
l
Thin rod about axis through center ⊥ to length
$I = \frac{Ml^2}{12}$
Axis
l
Thin rod about axis through one end ⊥ to length
$I = \frac{Ml^2}{3}$
Axis
$2R$
Solid sphere about any diameter
$I = \frac{2MR^2}{5}$
Axis
$2R$
Thin spherical shell about any diameter
$I = \frac{2MR^2}{3}$
Axis
R
Hoop about any diameter
$I = \frac{MR^2}{2}$
Axis
Slab about ⊥ axis through center
a
b
$I = \frac{M(a^2 + b^2)}{12}$

4.6 Introduction to Statics: what is required for something to be immobile?

Statics is all about making sure that buildings and machines are stable. Most engineering students start with a full course dedicated to statics, we summarize in a few days. Consider a bridge touching the ground in only two places. If we're on this bridge, we'd like it to be in *static equilibrium*. We don't want it to move. We originally learned that equilibrium is when $\sum \vec{F} = m\vec{a} = 0$, or the net force on an object is zero, so it's not accelerating. However, this isn't enough. What if there is an upward force on one side of the bridge and an equal downward force on the other side of the bridge? Then it would have *angular acceleration*. This wouldn't be good for us on the bridge even if $\sum \vec{F} = 0$. Consider the two blocks resting on the larger block at right. The one near the middle is stable because the normal force and the force of gravity are equal and opposite. How about the one hanging over the edge? The force of gravity provides a torque about the point of contact, rotating that block… *into the page*. In a moment, the block will roll off the edge, the normal force will be zero, and the acceleration will be $\vec{g}$.



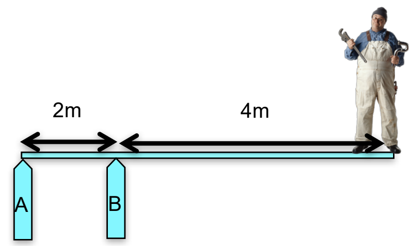
Thus, for static equilibrium, we add $\sum \vec{\tau} = I\vec{\alpha} = 0$, or the net torque on an object is also zero, so there's no angular acceleration, as well as no linear acceleration. We learned to use a dynamics lens with a protocol using $\sum \vec{F} = m\vec{a}$ equation where we would identify acceleration and forces and solve for the unknown. Now we have two *simultaneous equations* (force and torque), which we must keep separate – torque is not force, and force is not torque. With our two equations, we can solve for two unknowns such as the two unknown forces acting on each end of a bridge. Let's try it:



Exercise 1:

I am helping a 90 kg friend do some construction work by supporting one side of a plank for him to stand on. We want to know the force provided by pylon A and by my finger in order to keep the system in static equilibrium. The first question is, "what is the body that we are considering to be in equilibrium?" It's the plank that has forces and torques acting on it. But, the plank is static, so $\sum \vec{\tau} = I\vec{\alpha} = 0$, and $\sum \vec{F} = m\vec{a} = 0$. Please make a good free body diagram of all the forces acting on the plank and define the positive direction for forces (up or down), and for rotation (into the paper or out of the paper). Can you identify all the forces? Before you can identify all the torques, you need to define the center of rotation. Because $\vec{\alpha} = 0$, any point can be considered the center of rotation. However, the clever student will choose a point that is acted on by an unknown force because then the torque provided by this unknown force is zero, removing the unknown force from the torque equation… *lovely!* With only one unknown force in the torque equation, you can solve for this unknown in one step. So, pick either where the pylon or my finger is pushing on the board. You can solve for the other force by either using the force equation, or by using the torque equation again with the other "unknown force point" as the center of rotation. Please work this out carefully, and show that my finger provides 300 N, and pylon A provides 600 N of force.

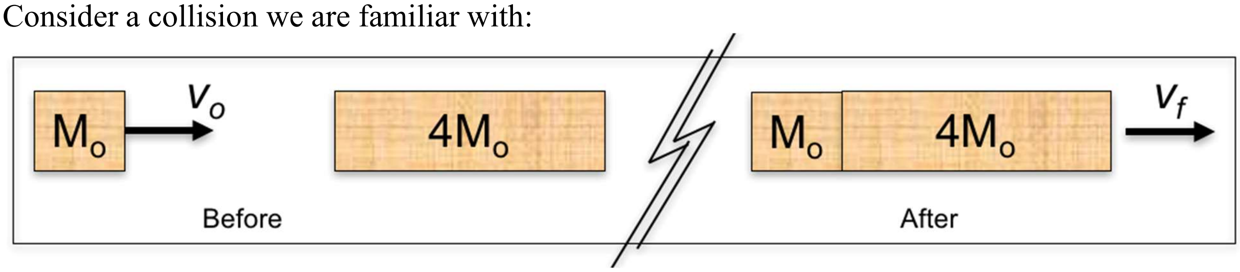
Exercise 2:
Please repeat the above calculation for this different geometry, where my friend (mass 90 kg) stands at the end of a very light rigid board secured to two pylons as shown at right.



- Please draw and label your own free body diagram, don't just use the diagram I provide.
- Find the force magnitude *and* direction that each pylon applies to the board.
- Go back and verify that $\sum \vec{\tau} = I\vec{\alpha} = 0$, and $\sum \vec{F} = m\vec{a} = 0$.

4.7 Introduction to Angular Momentum: what is conserved when bodies interact?

While we recognize Newton's second law: $\sum \vec{F} = m\vec{a}$, we also introduced force as a *single* interaction between two bodies whereby momentum is exchanged: $\sum \vec{F} = \frac{d\vec{p}}{dt}$, or a force causes the change in momentum of each object in opposite ways. Thus if $\sum \vec{F}_{system} = 0$, then $\frac{d\vec{p}}{dt} = 0$, and we learned that momentum of a system is conserved *if the sum of external forces is zero*.
Now we take the same steps with rotation dynamics. We know that
$\sum \vec{\tau} = I\vec{\alpha}$, but also
$\sum \vec{\tau} = \frac{d\vec{l}}{dt}$, *torque* changes the *angular momentum*; equals angular momentum's rate of change.
if $\sum \vec{\tau} = 0$, then $\frac{d\vec{l}}{dt} = 0$, *angular momentum* doesn't change… is conserved when there is no external torque.
Consider a collision we are familiar with:

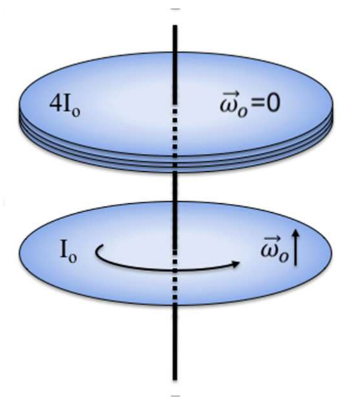


A block runs into and sticks to a stationary block 4 times its mass. The resulting body, at $5M_o$ moves off at $v_f = \frac{v_o}{5}$ by conserving momentum. In the collision, the smaller block receives an impulse to the left, and the larger object receives the same magnitude impulse to the right. Additionally, we can see that in this process we've lost 80% of the kinetic energy to heat. It's easiest to see if we express kinetic energy as:
$E_K = \frac{p^2}{2m}$, because we know that momentum is the same before and after, but the moving mass increased by a factor of 5 in the collision. Let's do this for a rotational collision.

A turntable rotates freely on a low friction bearing. It would rotate for a long time like this because there's no torque acting on it. But then, we drop four identical nonrotating disks on it.

Exercise 1:

The base of a turntable is rotating freely at rotational velocity ω_o on a very good bearing when four identical disks are dropped onto it. Please find:



a) The final rotational velocity. Please support your answer with the correct concepts and formulas and pay particular attention to which lens you use. Develop this carefully.

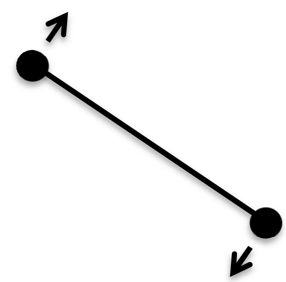
b) Is kinetic energy conserved? If not, how do you know? If so, how much energy is converted and what is it converted into?

c) What is the angular impulse $\Delta\vec{l}$, received by each body. Please include direction.

d) Consider the interaction whereby the stationary disks speed up and the rotating disk slows down. Can you identify an interaction that may generate heat?

Exercise 2:

Two identical bodies are tied together with a string, are spinning in space about the center at angular speed, ω_i, when a motor at the center pulls them both inward such that the final diameter of their paths is 1/3 the original diameter, or, d => 1/3 d_i. If this is in outer space, we can be sure there are no outside forces. We might consider conserving energy and/or angular momentum.



a) Is it possible for the angular momentum to change? If so how?

b) Is it possible for energy to change? if so, where did the energy go or come from?

Is it possible to conserve both angular momentum ***and*** energy? Let's find out!

c) What happens to the moment of inertia with this change? I => ___ I_i

d) If we conserve angular momentum what should be the new angular velocity? ω => ___ ω_i,

e) Would this change the kinetic energy? If so, by what factor: KE =>___ KE_i

f) Can we conserve angular momentum ***and*** energy? If not, which one must have changed, and where did that change come from?