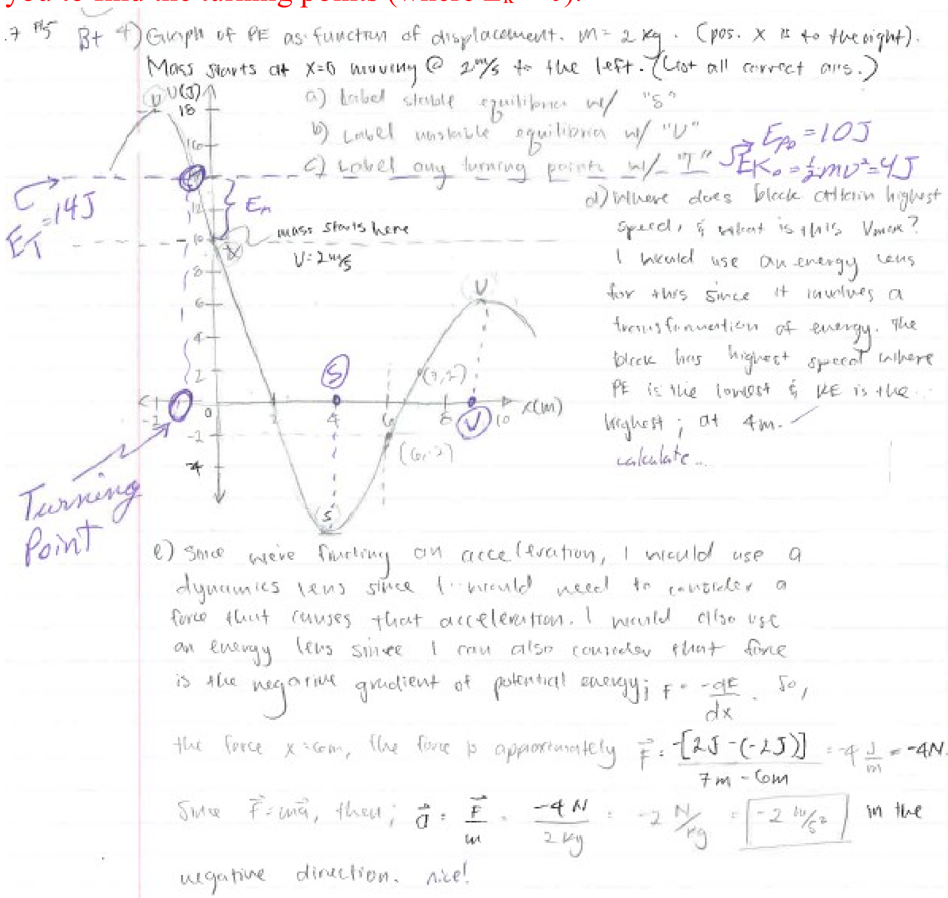
Problem Set #4 due beginning of class, Monday Oct. 14. Please state the lens you are using and why. Remember that you are graded on your ***communication*** of physics understanding.

1. Exercise 5 in 2.7, potential energy graph. The first thing you want to do with potential energy graphs is find the total energy = $E_P + E_K$. Draw this line in (as you see the student below did). This will give you the kinetic energy at all points (the difference between the total energy and the potential energy) and allow you to find the turning points (where $E_k = 0$).



2. An object starts at 10 m with a speed of 5 m/s and has an acceleration of – 4 m/s^2 + 2 m/s^3(t). Find the velocity and position after 3 seconds.

This is a straight kinematics lens because we're given and need to find motion as an explicit function of time. We recognize that a = dv/dt and v = dx/dt, so we have to integrate acceleration to get velocity and integrate velocity to get displacement:

$\Delta v = -4$ m/s^{2}t + m/s^3(t^2)… but at t = 0, the speed is 5 m/s so we have to add 5 m/s as the "integration constant" v(t) = 5 m/s – 4 m/s^{2}t + m/s^3(t^2)…

$\Delta x = 5$ m/s(t) – 2 m/s^{2}t^2 + (1/3) m/s^3(t^3)… but at t = 0, the position is 10 m so we have to add 10 m as the "integration constant": x = 10 m + 5 m/s(t) – 2 m/s^{2}t^2 + (1/3) m/s^3(t^3)…

At 3 seconds, I'm getting v = 2 m/s, and x = 16 m direction?

(5) $v_0 = 5\,m/s$ $a = -4\,m/s^2 + 2\,m/s^3 t$

$V_f = ?$ $t = 3s$

$x_0 = 10m$ $x_f = 3$

$a(t) = -4\,m/s^2 + 2\,m/s^3(t)$ Kinematics lens: finding position and velocity as a function of time.

$V(t) = \int a(t)dt$

$= \int (-4\,m/s^2 + 2\,m/s^3(t))\,dt$

$= -4\,m/s^2(t) + 1\,m/s^3(t^2) + C_v$ $C_v = V_0 = 5\,m/s$

$= -4\,m/s^2\,t + 1\,m/s^3(t^2) + 5\,m/s$

$V(t) = -4\,m/s^2(t) + 1\,m/s^3(t^2) + 5\,m/s$

$\boxed{V(3s) = 2\,m/s}$

$V(3s) = -4\,m/s^2(3s) + 1\,m/s^3(3s)^2 + 5\,m/s$

$= -12\,m/s + 9\,m/s + 5\,m/s$

$= 2\,m/s$

$X(t) = \int V(t)dt$ $C_x = X_0 = 10m$

$= \int (-4\,m/s^2(t) + 1\,m/s^3(t^2) + 5\,m/s)\,dt$

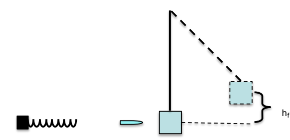
$= -2\,m/s^2(t^2) + \frac{1}{3}\,m/s^3(t^3) + 5\,m/s(t) + C_x$

$X(t) = -2\,m/s^2(t^2) + \frac{1}{3}\,m/s^3(t^3) + 5\,m/s(t) + 10m$

$X(3s) = -2\,m/s^2(3s)^2 + \frac{1}{3}\,m/s^3(3s)^3 + 5\,m/s(3s) + 10m$

$= -18m + 9m + 15m + 10m = 16m$ $\boxed{X(3s) = 16m}$

3. A loaded gun is cocked by compressing a spring of k = 10^4 N/m. and then releasing it behind a 20 g bullet. The bullet strikes and sticks inside of a 0.5 kg ballistics pendulum and swings upward to a final height of 50 cm. Presume the spring is massless and there is no friction in the system. Please find:



a) The bullet's speed.
b) how far the spring was compressed.
c) Does the bullet have constant acceleration in the gun, or does the acceleration change over time? Please explain your answer… identify a lens.
d) Please find the maximum acceleration of the bullet in the gun.
e) Did you identify the lenses at the very beginning, or one at a time for each question? Which do you think would be a better approach?

As soon as we see this, we are tempted to use an energy lens equating the initial spring potential energy to the final gravitational potential energy. However, the great majority of the bullet's kinetic energy is converted to thermal energy in the inelastic collision. Thus, we can find the kinetic energy of the bullet/block immediately after collision using an energy lens. However, to find the bullet's speed, we need to use the momentum lens because there is negligible outside forces so the momentum is conserved in the collision. The bullet's kinetic energy does come from the spring potential energy.

For letter "c" and "d", constant acceleration would be the result of a constant force. However, the force of the spring is proportional to the spring's compression. This the maximum acceleration would be when the spring is maximally compressed. This acceleration comes out to be 6000 times the acceleration of gravity, but so is the life a bullet! In fact, this acceleration is small compared to the acceleration the bullet experiences when it hits the target!

6). Lens: Energy Lens because energy is being converted from potential to kinetic and momentum

$k = 10^4$ N
$m_b = .02$ kg
$m_{box} = .5$ kg
$h_f = .5$ m

$E_{spring} \rightarrow KE_{Bullet} \rightarrow E_{therm} + KE_{Block} \rightarrow PE_g$

$\frac{1}{2}kx^2 \rightarrow \frac{1}{2}mv^2 \rightarrow \quad \rightarrow mgh$

$\frac{1}{2}(10000\,\tfrac{N}{m})x^2 \rightarrow \frac{1}{2}(.02\,kg)v^2 \quad \rightarrow (.5\,kg)(.5\,m)(10\,m/s^2)$

units

$\frac{1}{2}mv^2 \rightarrow 2.6\,J$

inelastic collision of bullet / box — lens? $\vec{p}$ because $F_{outside} = 0$

$\frac{1}{2}(.52\,kg)v^2 = 2.6\,J$

$m_b v_b = m_{box} v_{box}$

$v = \sqrt{\frac{5.2\,J}{.52\,kg}}$

$(.02\,kg)v_b = (.52\,kg)(3.16\,m/s)$

$v_{box} = 3.16\,m/s$

$v_{bullet} = \frac{(.52\,kg)(3.16\,m/s)}{(.02\,kg)} = \boxed{82.16\,m/s}$ (a)

Energy lens, because $E_s \Rightarrow E_K$

$\frac{1}{2}(10000\,\tfrac{N}{m})x^2 = \frac{1}{2}(.02\,kg)(82.16\,m/s)^2$

$x = \sqrt{\frac{(.02\,kg)(82.16\,m/s)^2}{10000\,N/m}} = \boxed{0.12\,m}$ (b)

$\left(\frac{kg\,\frac{m^2}{s^2}}{N/m}\right)^{\frac{1}{2}} = (m^2)^{\frac{1}{2}} = m$

Please include units.

c) The force of the bullet comes from the force of the spring ($F = kx$). To have a constant acceleration, there must be a constant force. The force of the spring is not constant because as the ~~displacement of compression change~~ spring reaches equilibrium, the F_{spring} decreases. Thus the bullet does not have constant a.

Nice!

d) $F_s = F_B = ma$

~~$\frac{1}{2}(10000)(.12)^2 = (.02)a$~~ ~~$a = 58000\,m/s^2$~~

E_p

$F = kx = 10000\,\tfrac{N}{m} \cdot 0.12\,m = 1200\,N$!

e) Lenses are the way to go! Yes!

$a = \frac{F}{m} = \frac{1200\,N}{0.02\,kg} = 60000\,\frac{m}{s^2}$

$= 6\times10^4\,m/s^2 = 6000$ gravities!

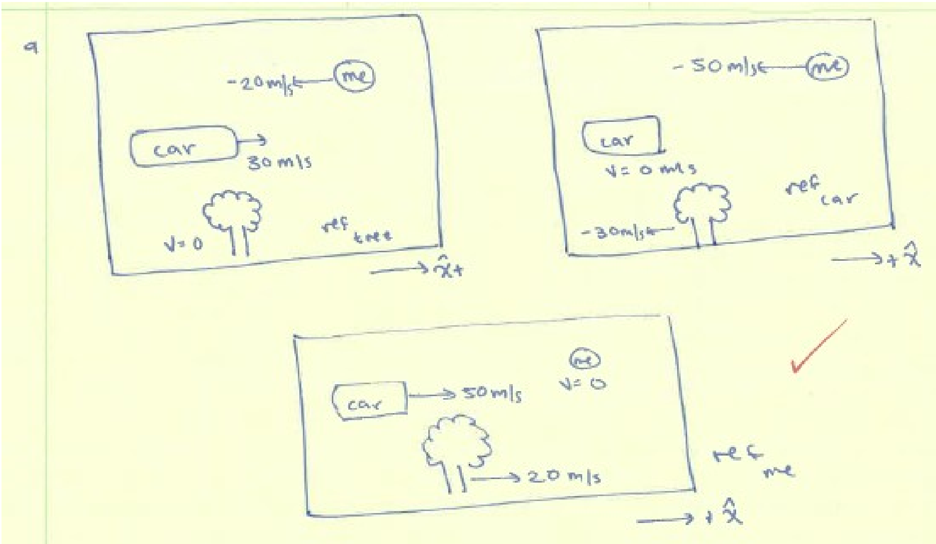
4. Using an energy lens, please show that if you drop a 5 kg box from 60 m, it hits the ground at ~35 m/s. But then, you *throw* the box *downward* from 60 meters height with an initial speed of 35 m/s.

a) Find the speed that it has when it hits the ground.
b) What if I throw it *upwards* at 35 m/s, what is the speed when it hits the ground?
c) What if I throw it straight off the cliff at 35 m/s horizontally, what speed does it have when it hits the ground now?
d) *Can* I throw a 5 kg box at 35 m/s? Please back up your answer.

I show in a video that if I double the energy, then the speed increases by root 2 or about 49 m/s. Also, conserving energy, it doesn't matter what angle I throw the box, the final kinetic energy (and speed) will be the same. We also see, using an energy lens, it is very unlikely I could throw 5 kg at that speed, requiring power and force from my arm that is really more than one would expect from me.

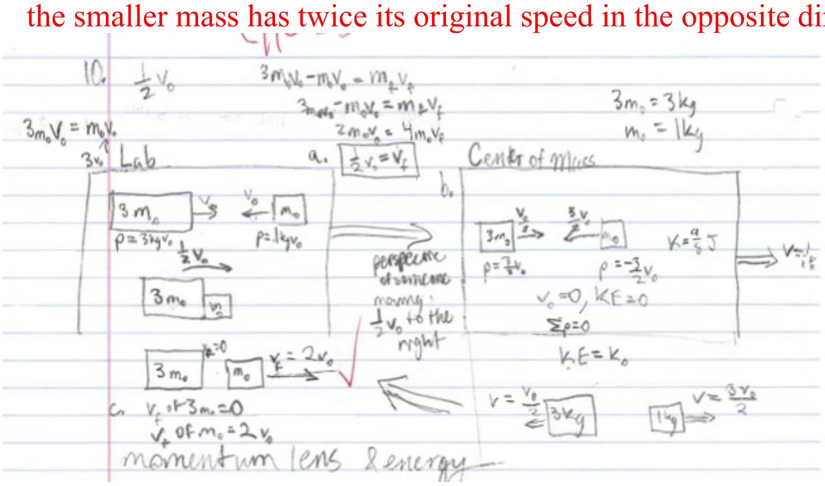
5. Exercise 1 in 3.0, changing reference frames

This is a simple kinematics lens because we are just looking at relative velocity. Each object sees itself at rest, but still sees the same relative velocity. For instance, in order to see itself moving at 0 m/s, the blue cart must add + 20 m/s to the velocity of each cart. Thus the tree has a velocity of + 20 m/s and the red bug has a velocity of + 50 m/s.



6. Exercise 2, in 3.1, What are the final velocities in this elastic collision?

We did this in class. We find that the velocity of the center of mass is $v_o/2$. We remember that we want to be in this reference frame because this is where one would see the system as having zero momentum. Thus the final momentum must also be zero. We should find that the larger mass is at rest after the collision and the smaller mass has twice its original speed in the opposite direction.



7. Dragsters have a mass of about 1000 kg and the best dragsters get to 44 m/s in about 0.8 s.
   a) What's the acceleration?

      This is straight kinematics because we have explicit descriptions about motion. The acceleration is 55 m/s^2, outrageously large… 5.5 gravities!!

   b) Estimate the coefficient of friction necessary to make this happen if you were in a regular car on flat ground.

      This is a forces (dynamics) problem because we have a force (friction) causing acceleration. The acceleration is outrageous, so the friction coefficient must be as well. First use a dynamics analysis in the y direction with a nice drawing where the acceleration is zero to find that the normal force = the force of gravity. You need a frictional coefficient of 5.5… impossible? Maybe. We'll see below that it really doesn't have to be that large.

   c) What's the average power output during this 0.8 s?

      This is an energy lens because we are looking at how the energy changes as a function of time, and the energy conversion is mechanical work (from the engine) to kinetic energy in the motion of the dragster. This is about 1.2 MW, or about 1600 HP… and outrageous amount of horsepower….. like 10 times as much as an average car. But again, dragsters aren't average. It was brought to my attention that this wasn't an adequate estimation: We calculated that this is the power the car received from the engine. However, the mechanical output of the engine was also turned into heat released from the spinning tires on the ground. We didn't include that. So, the engine must certainly be putting significantly more power than the 1600 HP we calculated. It's worth noting that if you don't spin your tires, there is little kinetic energy converted to heat, so you don't need to include this consideration.

d) Dragsters have their exhaust pipes pointed *upwards*, which ejects a huge amount of exhaust straight up into the air at very high velocity. What effect does this thrust have on the ability of the car to accelerate? *Why? Please start with clarification of reasons, drawings, lenses.*
   We use a dynamics lens looking at the forces in the y direction. Force is the rate of change of momentum of the heated exhaust upward, there is an equal downward force on the dragster because the force is between the dragster and the air. We can then examine the forces in the y direction on the dragster and realize that now the normal force must be equal to the force of gravity *and* this down force combined.

According to my calculations, the engines kick out about 18 kg of exhaust every second at about 230 m/s. This corresponds to a momentum change of 4400 kg m/s every second, exerting a force of 4400 N.

e) What is the momentum of this amount of gas?
f) How much force should this put on the vehicle? In which direction?
g) With this extra "downforce", what coefficient of friction is necessary in order to accelerate the dragster? Now, the normal force must be 14100 N, requiring a friction force of only 3.9, which is still very large, but more attainable.

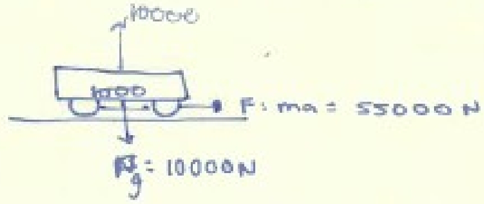
11. $m = 1000$ kg

best dragsters get to 44 m/s in .8 s

$F_f = \mu N$

a) $a = \Delta v / \Delta t = 44\text{ m/s} / .8\text{ s} = \boxed{55\text{ m/s}^2}$

Dynamics Lens because $\Sigma \vec{F} = m\vec{a}$



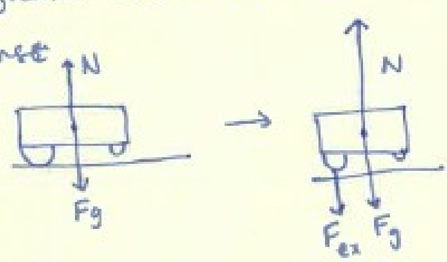
$55000 = (\mu)10000$

b) $\boxed{\mu = 5.5}$

c) ~~PE = W/t = 55000 / .8~~ $P = \dfrac{\frac{1}{2}(1000)(44)^2}{.8\text{ s}} = \boxed{1210\text{ kW}}$

d) When the exhaust ~~exerts~~ ejects exerts force downwards on the wheel, the normal force increases significantly, consequently increasing the force of friction which allows for greater acceleration

Dynamics Lens



e) $p = (18)(230) = \boxed{4140\text{ kg m/s}}$

f) $F = ma = (18)(230\text{ m/s}^2) = \boxed{4140\text{ N}}$ $\boxed{\text{downwards}}$

g) $55000 = (14140)\mu \Rightarrow \mu = \underline{\underline{3.89}}$

8. From Assessment #3

A- A

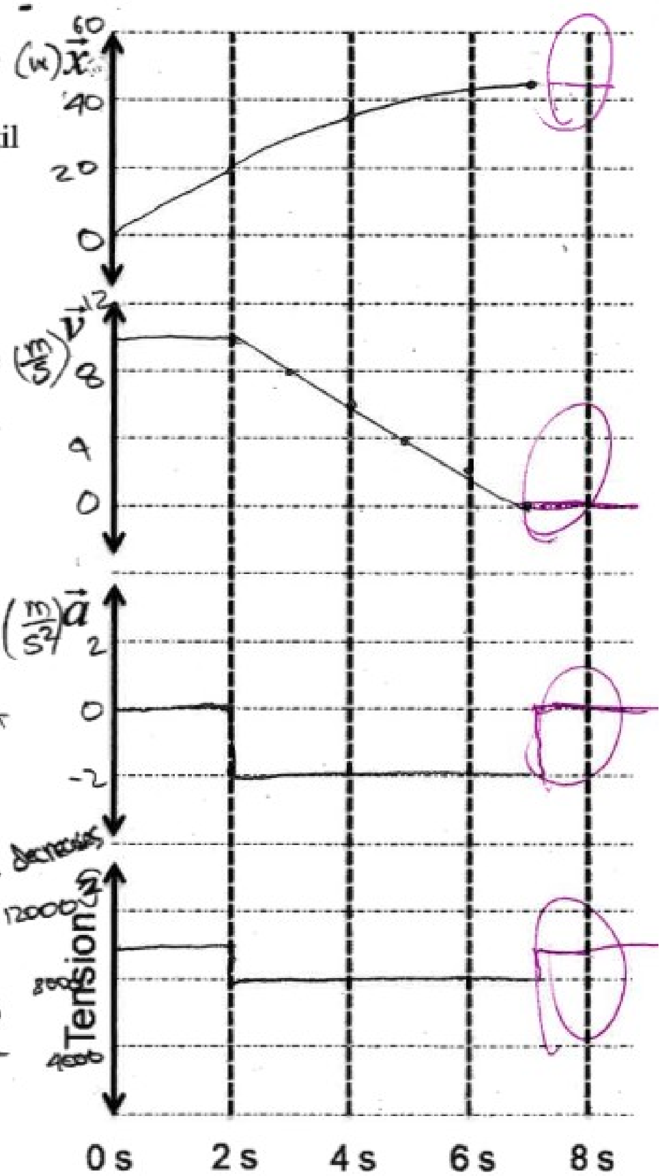
?

Assessment #3

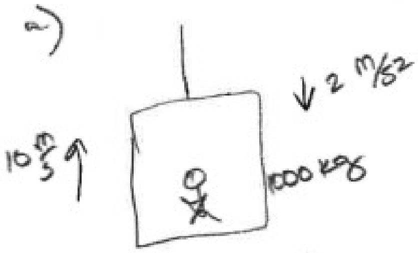
Imagine that you are traveling upward in an elevator at a constant rate of 10 m/s. after two seconds, you are slowing down at a rate of 2 m/s every second until you stop. The mass of the elevator is 1000 kg (with you in it).

a) Please graph the motion at right. Show work below

b) Please graph the tension on the cable over time. Please use the other side of this page to elaborate the beautiful work behind your answer.



a)



I will use a kinematics lens as it involves expressing motion as an explicit function of time.

$v = 10\ \frac{m}{s}$, constant for first 2 secs, then decreases

$a = -2\ \frac{m}{s^2}$, 0 for first 2 secs due to constant velocity

$\vec{x}$ = area under velocity curve

from 0-2s: $v_{avg} = \frac{\Delta x}{\Delta t}$

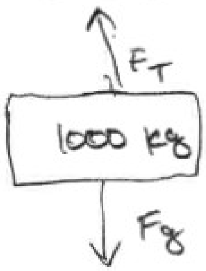
$\Delta x = v\Delta t = 10\ \frac{m}{s} \times 2s = 20m$

from 2-7s: $v_{avg} = \frac{\Delta x}{\Delta t}$

$\Delta x = v\Delta t = 5\ \frac{m}{s} \times 5s = 25m$

* assuming starting position is ground (0 m)

b) from 0-2 s:



$a = 0\ \frac{m}{s^2}$

I will be using a dynamics lens as I am looking at how the ~~sum~~ sum of forces cause acceleration. ?

$$a = 0\ \frac{m}{s^2}$$

$$\Sigma F = ma$$

$$\Sigma F = m \times 0\ \frac{m}{s^2} = 0\ N$$

$$\Sigma F = F_T + F_g$$

$$F_g = mg = 1000\ kg \times -10\ \frac{m}{s^2} = -10000\ N$$

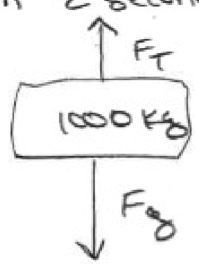
$$0 = F_T - 10000\ N$$

$$F_T = 10000\ N$$

ΣF diagram?

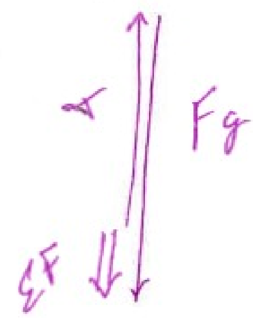
$a = 0$

from 2 seconds - 7 seconds (rest):



$\downarrow a = -2\ \frac{m}{s^2}$ $\quad \Sigma F = -2000\ N$

$a \Downarrow$



I will use a dynamics lens as it involves how sum of *which*? forces cause acceleration.

$$\Sigma F = ma$$

$$\Sigma F = 1000\ kg \times -2\ \frac{m}{s^2} = -2000\ N$$

$$\Sigma F = F_T + F_g \Rightarrow F_g = mg = 1000\ kg \times -10\ \frac{m}{s^2} = -10000\ N$$

#9, From Wednesday's Class Activity, the Ballistics Pendulum. At Pete's 4:00 class, Phil threw a 41g bean bag causing the 2.0 kg cooler hanging on a 67 cm string to swing back 7 cm. Please see the video https://www.youtube.com/watch?v=1feir-_KhEQ&feature=youtu.be

a) Please estimate the speed that Phil is able to throw the bean bag. Please outline your work nicely as if this were an assessment… the way you should approach all physics problems… and maybe all challenges in general.

I would initially call into force an energy lens because the initial kinetic energy of the ball turns to gravitational potential energy of the pendulum (or the cooler/ball system). However, if I think this through with a step by step diagram (which you will need to do in order to get full credit for this problem) I'd realize that there is an inelastic collision between the bean bag and the cooler so lots of kinetic energy is changed to potential energy. In this collision, I know that I can use a momentum lens… because in order to change momentum, you need force. The outside force acting on the system during the collision is zero… or at least very small compared to the normal force acting between the bean bag and cooler. So, I do know that momentum is conserved in this collision. Thus $v_{bb} = v_{CS}(m_{CS}/m_{bb})$. In order to find the speed of the cooler/bean bag system, we can use an energy lens because the initial kinetic energy of the cooler/bean bag system (immediately after the collision) is converted to gravitational potential energy as the system rises at the end. Using Pythagorean's theorem, we find that the cooler should have risen 3.7 mm = 0.0037 m, corresponding to an initial speed of 0.27 m/s. Conserving momentum, this would have happened if the speed of the bean bag had a velocity of 13.2 m/s or 31 miles per hour.

b) Use kinematics to find the speed of the bean bag (from the video)

….Just so you know, my calculations for a) and b) didn't match too well… Maybe I just made a mistake, or maybe physics is just wrong… but maybe there's some way to explain why this should be the case. We can talk a little about it Monday in class.

Just using a kinematics lens because the video gives us exquisite information about position as a function of time… for b), we know that $v_{ave} = \Delta x/\Delta t$. We can examine the video and see how many frames it takes the bean bag to move a certain distance. I counted 16 frames for the bean bag to move 1 meter, so I have $v_{ave} = \Delta x/\Delta t = (1\text{ m})/(16\text{s}/240) = 1\text{m}*240/16\text{s} = 15$ m/s, or about 33 miles an hour.