Problem Set #7 due beginning of class, Monday, Nov. 4

1) Chapter 5.4, Exercise 3, How fast are you going infinitely far from the earth?

2) at elevation!

Energy lens, You would need enough

$KE = PE_g$;

$\frac{1}{2}mv^2 = \frac{mM_e}{r}G \cdot \frac{r}{r}$; $\frac{1}{2}mv^2 = \frac{mM_e}{r^2}G$ ∴

$\frac{1}{2}v^2 = gr$

$v = \sqrt{2gr} = \sqrt{2 \cdot (10\,m/s^2) \cdot (6{,}400{,}000\,m)}$

$\boxed{V = 11{,}300\ m/s}$

• Energy lens still

$\frac{mM_E}{r}G$ ∴ $KE = KE + PE$

2) Chapter 6.0, carefully consider Examples 1, 2, and 3. Then do the following:
What if there is a coefficient of friction $(\mu_d = 0.1)$ on the 1 kg mass as is slides across the horizontal surface? *The system at right is dropped through 1 m. Even if the question doesn't give you a distance, you can always assume a certain distance.*

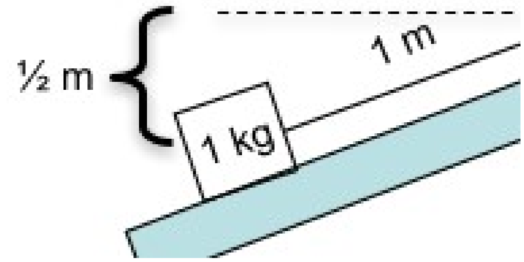
a) How would this change the energy considerations in Example 1? Find the new speed of the system as it hits the ground 1 m below. Then find the time to fall and the acceleration.

Using the energy lens, we realize that now some of the potential energy is changed to thermal energy as well as kinetic energy, reducing the final speed and the resulting acceleration:
$E_{gp} = E_k + E_{th}$. Using a dynamics lens, we look at the 1 kg mass, knowing it is in equilibrium in the y direction, so the normal force = mg = 10N. Thus, the frictional force is 1 N and 1 J of thermal energy is produced. The 0.25 kg block loses 2.5 J of E_{gp} so now only 1.5 J of kinetic energy is shared between the two blocks (corresponding to a final speed of about 1.5 m/s, and average speed of 0.75 m/s) as opposed to the 2.5 J without friction (corresponding to a final speed of 2 m/s, or average speed of 1 m/s). Corresponding times and accelerations of 1.3 s, and 1.2 m/s^2, and 1 s and 2 m/s^2 without friction.

b) How would this change the dynamics considerations of the system? Find the new acceleration directly (is it the same as you found above?) and tension in the string.

Using a dynamics lens we can write that $\sum \vec{F}_{system} = m_{system}\vec{a}_{system}$, Please make a good FBD with all relevant forces on the system (note that some forces don't matter like the tension in the string because it pulls in both directions, so it exerts no force on the system), label the + direction as to the right and down, so there are now two forces, + 2.5N and – 1N, so the total force is 1.5 N, resulting in an acceleration of 1.2 m/s^2 as opposed to 2.0 m/s^2 with no friction.

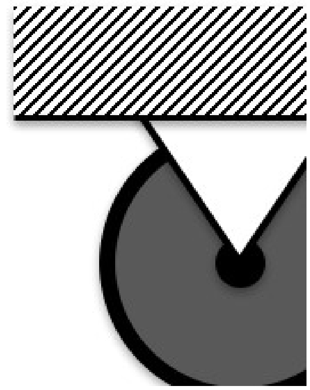
3) Consider the system at right where the 1 kg box is on a very slippery table inclined such that if the system moves one meter, the box changes elevation by half a meter.



a) How does all this change the energy balance equations you set up in Chapter 6.0? *Calling the positive direction with the 250g mass falling, the 1 kg mass would **gain** potential energy, to be added to the energy equation.*

b) Can you tell me which way the system will acceleration (if at all)? How can you be sure? *Because the 1 kg mass gains twice the E_{gp} that the 250g mass loses by falling, the system will accelerate with the 1 kg <u>falling to the left</u> at the same acceleration that the system accelerates to the right in the problem above (without friction).*

4) Chapter 6.1 Example 1



a) Using an energy lens, we know that the potential energy will decrease as the block falls, so it must accelerate downward. However, using a rotational dynamics lens, we know that there must be tension in the string in order to angularly accelerate the pulley, so there is an upward force on the block causing its acceleration to be less than gravity.

b) Using a dynamics lens, we look at the falling mass because forces cause it to accelerate downward. With a FBD, I see that the Tension < F_g because it is accelerating downward.

c) With an energy lens, the E_{pg} of the hanging mass turns to linear E_k of the hanging mass + rotational E_k of the wheel.

d) $a = r\alpha$, straight up kinematics lens.

5) Chapter 6.1 Example 2 *These answers are the same as for the system above in Example 1. The only difference is that falling mass is also the rotating mass.*

6) Chapter 6.1 Example 3 *Using and energy lens, we see that the loss of potential energy of the hanging mass transfers to the kinetic energies: Linear E_k of the hanging mass and rotational E_k of the wheel. Because B has a smaller radius pulley it will have a larger ω/v ratio, and thus a larger ratio of rotational E_k to linear E_k. Using a kinematics lens, "A" will have a higher linear speed, shorter fall time, larger linear acceleration. Using a dynamics lens, we can show that $T_A < T_B$ because the larger acceleration means a larger net force, which requires a smaller tension.*

7) You won't need to integrate to find center of mass on midterms or final exam. What if the diving board in chapter 6.2 was <u>*not*</u> of uniform thickness, but was thicker at the left end that attaches to pylon A (see image). Imagine that the board smoothly increases from the free end at right, and is three times the thickness (and linear mass density in kg/m) at left than at right. Then where would be the center of mass of the 6 *m* board? You'll use some calculus for this.

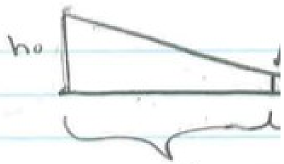
- The first conceptual step will be to put the thickness of the board (and therefore the mass density) as a function of displacement. With the pylon at x = 0, what linear function would define the thickness of the board to be h_o at the origin and $h_o/3$ at x = 6 *m*?

You learned this stuff in like 6th grade I think, but never thought of using this math: y=mx+b. We know that mass = Volume x density, and that V = length x height x width. However, because we assume that the width of the board is constant, we will drop the width from the equations and say that mass = Area x density. We have to do several calculations:

1) *find the area of the board, which we later use to find the density of the board*
2) *find the formula for the thickness of the board as a function of x, so we can integrate it.*

3) *Recognize that torque = mg*r, and express it in terms of x, so we can integrate it.*
4) *Execute the integration and compare the torque with what we know it must be: $t = mg*x_{cm}$ and solve for x_{cm}.*

(a)



$y = mx + b$

$b = h_o$

$m = -\frac{h_o}{9m}$

$y = -\frac{h_o}{9m} x + h_o$

com

$V =$

$V =$

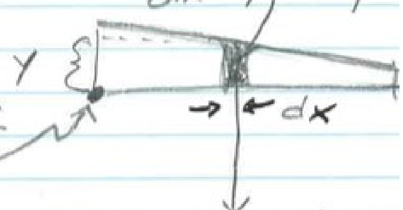
$V = ($

$V = ($

$V = (4$

$m = (4m$

$dm = y \cdot dx \cdot \rho$



center of rotation

origin.

$F_g = (dm)g$

total $\tau_g = \rho g h_0 \left[-\frac{x^3}{27 m} + \right.$

$= \rho g h_0 \left[-\frac{216 m^3}{27 m} \right.$

$\tau_g = (10 m^2) \rho g h_0 \overset{?}{=}$

This the definit

from before, we know m

Mass $= (4 m) h_0 \rho$ $\quad \rho = \frac{}{(4}$

meters

2.5m

$\tau_g = (10 m^2)$ mass $g \cancel{h_0} =$

3m is the center of the board <u>*length*</u>

- How far from pylon A is the center of mass of the board?

We find 2.5 m from the base of the board, consistent with what we'd expect knowing that for a uniform thickness board, it would be at 3 m.

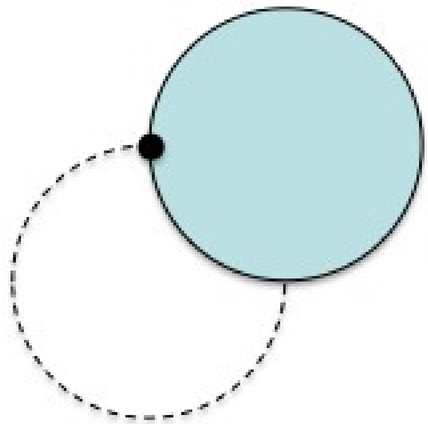
- What about the design of the board tells you that "A" exerts a downward force and "B" exerts only an upward force?

Note that one end of the board is bolted down, and the other point is simply resting on a support.

8) Chapter 6.3 Example 1 *You could solve this with the 2 simultaneous equations resulting from a dynamics and a rotational dynamics analysis of the rotating mass. However, you can also solve it with a single equation if you look at the point of contact as the center of rotation (which it is at any given moment). The torque = mgr, but you have to consider the full moment of inertia using the parallel axis theorem: ½$mr^2 + mr^2$, resulting in an angular acceleration of 2/3 * g/r. We can use kinematics to see that this means the acceleration is 2/3 g, which seems reasonable because it's less than gravity because there is tension on the string pulling it upward, providing the torque to angularly accelerate it. To find the tension, we can use the dynamics lens because the forces are accelerating the yo-yo. We should find that the tension in the string is mg/3. Lastly, we can check this by looking at the center of the disk as the center of rotation. Now the torque is provided by the tension in the string and the moment if inertia is just that of the disk: ½mr^2. Again, we get an angular acceleration of 2/3 * g/r… just as before, so we are happy.*

9) Infamous Tow Truck Problem: A 2-ton Tow Truck pulls a 1-ton car on a smooth level road, with a rope that has a tension of 3000 N on it. If the wheels of the car are free to roll, what coefficient of friction is necessary between the Tow Truck's wheels and the ground? This is a multi-step problem that will require some thought and some drawing. *This is a dynamics problem through and through because the force of friction on the wheels is accelerating the entire 3-ton system, and the tension in the rope is accelerating the 1-ton car, and the tension and friction are accelerating the 2-ton tow truck. It's up to you to make a good FBD and know which of the systems to look at for each one. It's important to recognize that the tension is an internal force and thus provides no force to the system. Examining the 1-ton car alone, we find that the acceleration must be 3 m/s^2. Examining the 3-ton system, we see that the force of friction accelerating the system must be 9 kN. The necessary coefficient of friction must be 0.45.*

10) Please thoroughly read 6.3 first. A disk of uniform mass distribution, total mass m_o, and radius R, is secured to a wall with a low friction pivot that allows rotation as shown at right. The disk is started in the higher position where its center is the same height as the pivot and is allowed to drop and swing.

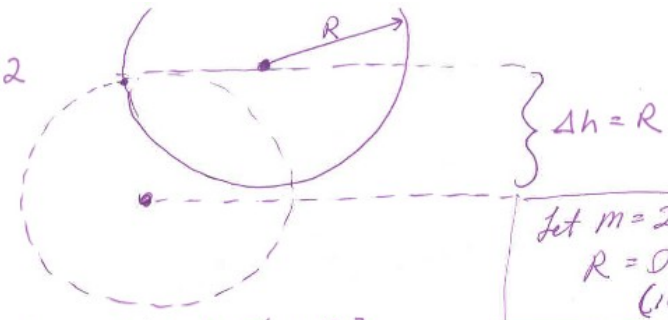


- The moment I let the disk go from the upper position the center of mass accelerates downward. Is a <, =, or > g? *Using a dynamics lens, we can consider what would happen if I just dropped the disk… it would accelerate downward at g. In this case, the pivot must provide an upward force to prevent that point from accelerating downward. So, the total force on the object is now < mg, and the acceleration must be < g.*
- Please find the angular acceleration of the disk the moment I let it go and from this information find the downward acceleration of the center of mass of the disk. *This is the same as #7 above. We should get 2/3 g/r*
- Please find the force the pivot point provides to the disk the moment I let go of the disk. *Again, as above, it should be mg/3*
- Does the angular acceleration of the disk remain constant as the disk falls to the lower position? How do you know? *Using a dynamics lens, we can see that everything changes as the disk rotates downward. The force of gravity is no longer perpendicular to the radius, so the torque must decrease.*

We want to find the force on the pin when the disk is at the bottom location. In order to solve this complicated, multidimensional problem, please consider:

- What is the complete energy transition happening as the disk rotates from top to bottom?
- What is the complete dynamics going on when the disk is at the bottom of the swing? Is the force on the pivot just equal to mg? Why or why not?
- Find the force on the pivot when the disk is in full swing at the bottom. Include direction.

#2



Let $m = 2\,kg$
$R = 0.1\,m$
(10 cm)

Uniform disk, $I = \frac{1}{2} m R^2$

as this swings down, I know that $PE \Rightarrow KE$
if Friction $= 0$, no energy is converted to
thermal energy.

KE = Translational + Rotational energy.
so now we have 2 unknowns: ω, v, but
$v = \omega R$, so we're OK.

OR you can view the pivot as the
center of rotation and just say
$PE \Rightarrow KE_{rotation}$ But you have to use
the // axis theorem to calculate I
about the rotation pt.

Either way, I get $\omega = \left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}}$
or, if $R = 10\,cm$ $\omega \approx 11/s$ This is almost
2 times around
per second!

#2b $\vec{L} = I\vec{\omega}$, but I need to recognize that we are not rotating about the center of mass, so $I = MR^2 + I_{cm} = \frac{3}{2}MR^2$

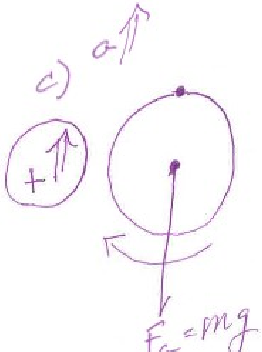
if $m = 2kg$ $\quad I_{cm} = \frac{1}{2}mR^2$

$I = 0.003 \, kg \, m^2$

$$L = \frac{3}{2}mR^2\left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}} = (3m^2R^3g)^{\frac{1}{2}}$$

or, if $m = 2kg$ + $R = 0.1m$, $L \approx 0.35 \, kg \frac{m^2}{s}$

c)



$\omega = \left(\frac{4}{3}\frac{g}{R}\right)^{\frac{1}{2}}$ This is dynamics as there is a_c ⇑ caused by the $\Sigma\vec{F}$

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^{\cancel{2}}}{\cancel{R}} = \omega^2 R$$

$$= \frac{4}{3}\frac{g}{\cancel{R}}\cancel{R} = \frac{4}{3}g$$

because we have uniform circular motion, a_c is up ⇑. F_g is down ⇓, so F_{pivot} ⇑

$\Sigma\vec{F} = m\vec{a}_c$

$F_p - F_g = ma_c$

$F_p = ma_c + F_g$

$= ma_c + mg$

$= m\left(\frac{4}{3}g + g\right) = m\left(\frac{7}{3}g\right)$

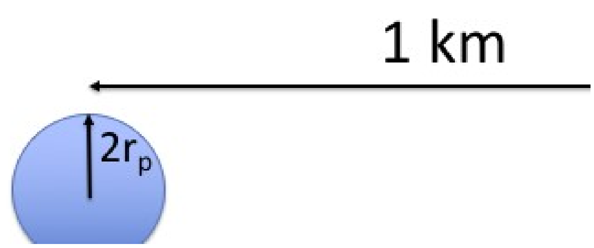
if $m = 2 \, kg$ $\quad F_p \approx 47 \, N$ ⇑ The key here is that the force is $> 2 * F_g$!

11) You want to put yourself in a place between two planets (1 km apart) where you will not feel any gravitational force. The planets are made of the same substance, but one has twice the radius of the other.

a) How far from the larger planet should you put yourself?

b) Find the position of the center of mass of this two-planet system.

c) Comment on how realistic this scenario is.



10) $F_A = F_B$

$F = \frac{m_1 m_2}{r^2} G$

$m \propto \text{Volume} \propto r^3$

$m_A = 8\, m_B$

$F_g = \frac{m_1 m_2}{r^2} G$ $\times \underline{\underline{8}}$

$r_A^2 =$

$r_A = \sqrt{8}$

$3.8\, r_B \approx 1\ \text{km}$

$r_B \approx \underline{\underline{0.26\ \text{km}}}$ $\quad r_A \approx 0.$

b) C.M. the planet

"Balance" on this point

C) yes, gravity would accelerate the masses toward each other, in order to not have yourself squished between the two planets, they would have to be rotating about their center of mass. Then the gravitational force would just accelerate them centripetally. However, then you would also have to rotate about this center of mas to remain in between them. However, then you would not be in equilibrium but would require a force pulling you toward the center of mass (and the bigger planet) in order to remain in a stable rotation. Thus, your new stable place would have to be closer to the larger planet than 0.26 km so that the force pulling you toward the larger planet accelerates you centripetally to remain rotating with the planets.