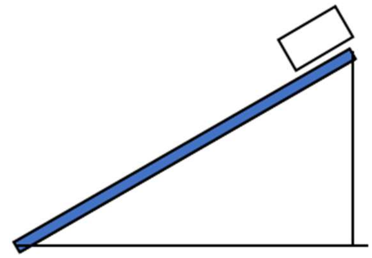
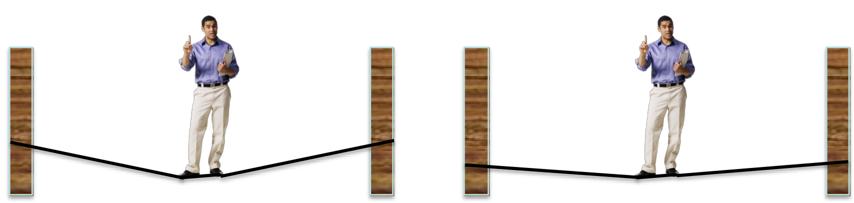
PS#9 Due in Class Monday, Nov. 18. Please pay good attention to describe the lens you are using and explain your method.

**** Make sure to consider the direction of acceleration to inform your choice of axis. Do you remember how to pick a good axis?

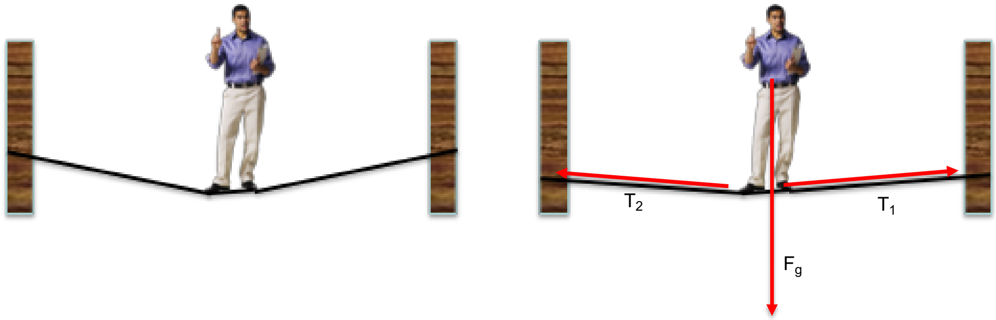


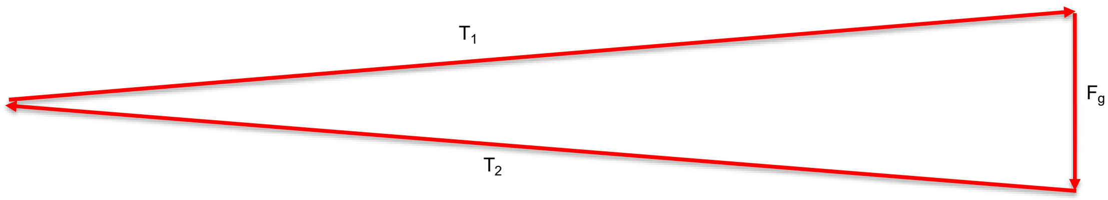
1. Please read section 7.2 and consider the cart of mass m_o at right, released from rest on a low friction surface.
   - Please find the resultant force on the cart in terms of constants that we know. Clearly outline your approach. We recognize this as a dynamics problem because the forces cause acceleration down the ramp. We do a good free body diagram and note that the interesting directions are parallel and perpendicular to the inclined surface because the cart accelerates parallel to this surface. We decompose the force of gravity into a parallel component and a perpendicular component. We see that the parallel component is about half of the full force of gravity or about ½mg.
   - Please estimate the acceleration down the track. Recognizing that F=ma, the acceleration down the ramp is about half of gravity, or about 5 m/s^2.
   - Repeat the above two questions if there is a coefficient of dynamic friction of 0.3 between the cart and the track. We need to find the normal force. If we did a good job with the FBD, we should see it's about 0.85 mg, yielding a force of friction of about 0.25 mg. Assuming the cart is moving downward, this force of friction is up the ramp, leaving only about 0.25mg down the ramp, for an acceleration of 2.5 m/s^2.
   - What coefficient of friction would be necessary for the cart to move at a constant speed? Constant velocity means it's in equilibrium and the sum of the forces = 0. So, the force of friction needs to be about ½mg. Given a normal force of about 0.85, the frictional coefficient would need to be about 0.6.
   - If the block had wheels with considerable mass, how would this affect the acceleration? Why? I'd prefer to use an energy lens because when we add mass to the wheels, we'd see that in the energy transformation from potential energy to kinetic energy, some linear kinetic energy would be sacrificed to provide rotational kinetic energy of the wheels. Using a dynamics/rotational dynamics lens, we realize that in order to rotationally accelerate the wheels, torque is necessary requiring tangential force on the wheels in the "uphill" direction… this force decreases the acceleration of the cart.
2. Slacklining s pretty fun, but you have to run some webbing between two trees first. Below, you see two pictures of me at 70 kg, slack lining.



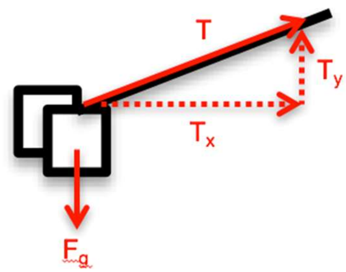
   a) In which drawing is the line tighter? Please prove how you know this with a good force drawing and discussion. Lens? Using a dynamics lens… statics. If the vertical acceleration = 0, then the sum of the vertical forces = zero. So, the force of gravity = the vertical components of the tension. Thus, the strings that are more horizontal must have a greater tension. Please see how I have drawn the sum of the forces diagram below for the scenario at right. Notice that the tension is 5 to 6 times as great as the force of gravity. So for the figure at right, the tension is about 4000 N.

b) Using your force drawing, please estimate the tension on the slack line at left.





3. You are watching the fuzzy dice from the rearview mirror. As you take off on level ground, it makes an angle as shown at right.



a) state how you will inform your choice of axis.

This is a dynamics lens because we have forces acting on the dice causing them to accelerate. We write $\sum \vec{F} = m\vec{a}$ and identify forces and direction of acceleration. Because the acceleration is horizontal, we break things up into horizontal and vertical components.

b) Estimate the acceleration of the car.

We see that $T_y = F_g$ because $a_Y = 0$. This we can see that the x component of tension (which is what's accelerating the dice in the x direction) is about twice $3*T_y = 3*F_g$ So the acceleration must be about 3 gravities or 20 m/s^2.

c) What must be the coefficient of friction of your tires for this to happen?

Because the normal force on the car = F_g between the car and road (because there's no vertical acceleration), we can see that the coefficient of friction would have to be an extraordinarily high value of 3.0 in order to attain such a high acceleration.

d) Is this realistic?

It's possible, but very unlikely, and not possible for regular tires and cars.

e) If the mass of the dice is 100 g, what is the tension in the string?

Looking back at the work we did for b) we can see that the tension should be a little more than $3*F_g$ or about 35 N.

4. Consider the fuzzy dice above. Now the car is stationary and you are sitting it in. You grab the dice and pull them to one side exactly as in the diagram above. Then you let go of them.

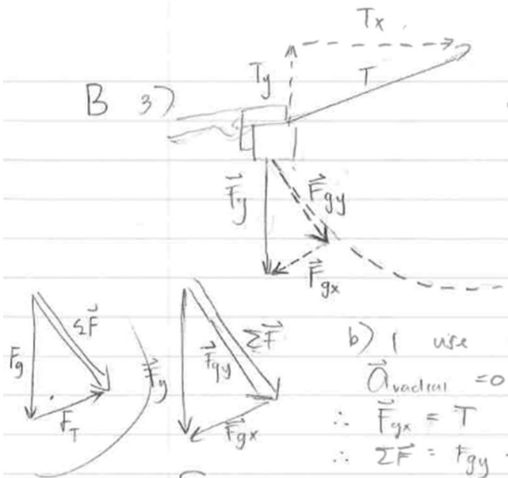
a) **** Choose a good axis. Is the direction of acceleration the same as above? State how this direction will inform your choice of axis.

Now the acceleration is tangential, perpendicular to the radius, so our axes are radial and tangential, so we keep tension as a single force and decompose gravity into parallel (tangential) and perpendicular (radial) components.

b) Again find the acceleration of the dice with direction.

c) Again, if the mass of the dice is 100 g, please find the tension in the string. Is it the same as the string above? Why might this make sense?

B 3)



a) I use a dynamics lens since I see forces causing acceleration and since I see a body moving in a circular path; it has a_{cp} caused by some force; The $\vec{a}$ this time is in the direction of $\Sigma\vec{F}$ along the circle

b) I use the same lens for the same reason.

$\vec{a}_{radial} = 0$

$\therefore \vec{F}_{gx} = T$

$\therefore \Sigma\vec{F} = F_{gy}$

$F_{gy} \sim 0.9 F_g = 0.9(mg)$

$\Sigma\vec{F} = m\vec{a} = F_{gy}$

$0.9 \cancel{m}g = \cancel{m}\vec{a}_{tang}$

$\vec{a} = 0.9g$

$\boxed{\vec{a} = 9 \ m/s^2}$ good

c) I use the same lens for the same reason.

I don't understand

$T = F_{gx} \sim 0.4F_g$

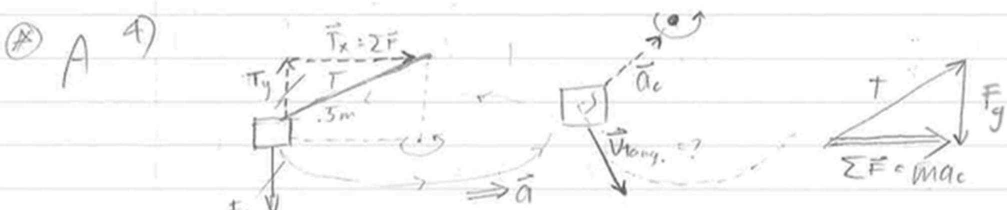
$T = 0.4(mg)$

$T = 0.4(0.1 \ kg)(10 \ m/s^2)$

$\boxed{T = 0.4 N}$

5. Consider the fuzzy dice above. Now you are holding them from the end of the 50 cm string, and spinning the dice around in a circle. The path of the dice is a circle in the horizontal plane. Estimate the speed of the dice and the tension in the string.

A 4)



I use a dynamics lens since I see forces causing acceleration. $\Sigma\vec{F} = m\vec{a}$.

$\vec{a}_y = 0 \Rightarrow F_g = T_y$

$T_x \sim 3T_y$

$\therefore T_x \sim 3F_g$

$\therefore T_x = \Sigma\vec{F}$

$T_x = \Sigma\vec{F} = m\vec{a}_c = m\frac{v^2}{r}$

$3F_g = m\frac{v^2}{r}$

$3\cancel{m}g = \cancel{m}\frac{v^2}{r}$

$3gr = v^2$

$\vec{v} = \sqrt{3gr}$

$\vec{v} = \sqrt{3(10 \ m/s^2)(0.5 \ m)}$

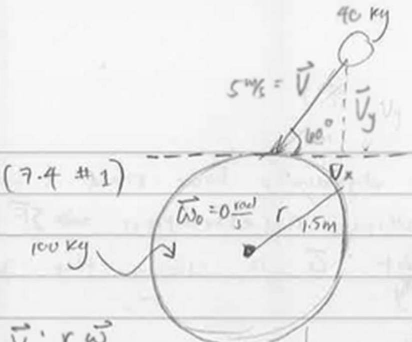
$\boxed{\vec{v} = 3.87 \ m/s}$

6. 7.4 Exercise 1, a child jumps onto a carousel.

$\vec{V}_x = (5\ m/s)\cos 60$ $\vec{V}_y = (5\ m/s)\sin 60$

$\vec{V}_x = 2.5\ m/s$ $\vec{V}_y = 4.33\ m/s$

A-7) (7.4 #1)



a) I use an angular momentum $\vec{L}$ lens since I see that w/ no external torques, then $\vec{L}$ is conserved.

$\Rightarrow \Sigma \Delta\vec{\tau} = 0 \Rightarrow \vec{L}_i = \vec{L}_f$ $\vec{L} = I\vec{\omega}$

$\vec{L}_{child} + \vec{L}_{disk} = \vec{L}_{child+disk}$

$m_c r \vec{v}_x + (\frac{1}{2} m_d r^2)\vec{\omega}_0 = (m_c r^2 + \frac{1}{2} m_d r^2)\vec{\omega}$

$\vec{\omega} = \dfrac{m_c r \vec{v}_x}{m_c r^2 + \frac{1}{2} m_d r^2} = \dfrac{(40\ kg)(1.5\ m)(2.5\ m/s)}{(40\ kg)(1.5\ m)^2 + \frac{1}{2}(100\ kg)(1.5\ m)^2}$

$\vec{\omega} = \dfrac{150\ kg\ m^2/s}{202.5\ kg\ m^2} = \boxed{0.74\ rad/s}$

$\vec{V} = r\vec{\omega}$

$\vec{\omega} = \frac{\vec{V}}{r}$ $\vec{L} = I\vec{\omega}$

$\vec{L} = mr^2(\frac{\vec{V}}{r}) = mr\vec{v}$

$I_{point\ mass} = mr^2$ $I_{disk} = \frac{1}{2}mr^2$

☆ b) I use an energy lens since I see transformations of energy as the child jumps onto the carousel. $\Sigma E_i = \Sigma E_f$.

$\Sigma E_i = KE_{child} = \frac{1}{2}mv^2 = \frac{1}{2}(40\ kg)(5\ m/s^2) = 500\ J$

$\Sigma E_f = KE_{child+disk} = \frac{1}{2}[(40\ kg)(1.5\ m)^2 + \frac{1}{2}(100\ kg)(1.5\ m)^2](0.74\ rad/s) = 75\ J$

oops, while this student did a great job, there is a little mistake here. The final kinetic energy is $E_{K-Rot} = \frac{1}{2}I\omega^2, where\ I = \frac{1}{2}m_{disk}R^2 + m_{girl}R^2$. This student forgot to square the rotational velocity. I get a final rotational kinetic energy of only 55 J., We see that close to 90% of the kinetic energy is transformed to thermal energy. Also, please note that although the kinetic energy is correctly calculated, the square for the speed needs to be outside the parenthesis.

Angular momentum lens

c) The collision would decrease the rotation rate, since the carousel is rotating in the opposite direction that the child would hit and provide angular momentum.

$$\vec{L}_o = \vec{L}_f$$

$$mvr_\perp - I\vec{\omega} = L_f$$

d) momentum lens. Momentum is always conserved since there are no outside forces. The girl's momentum is transferred to the Earth, but since the Earth's mass is so large, velocity is negligable.

e) momentum lens because no outside forces.

$$p_o = p_f$$

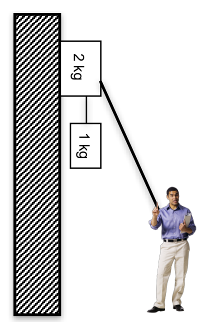
$$\cancel{p_{co}}^{\,0} + p_{po} = p_f$$

$$mv_{po} = m_{c+p} v_f$$

$$40kg(5 m/s) = 140kg\, v_f$$

$$v_f = 1.4 m/s \text{ southwest}$$

Also, again note that momentum is conserved (*all components* of momentum) independent of conserving the angular momentum (where only the tangential component contributes).

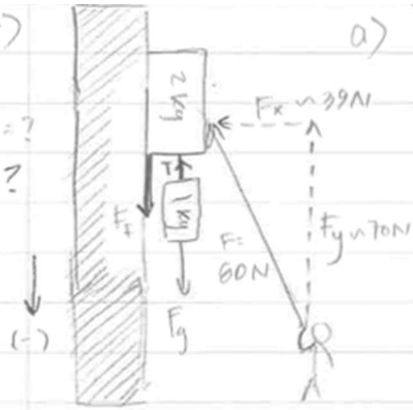


7. Please see a picture of me pushing a system of masses up a vertical wall with a coefficient of friction of 0.4. If I push the stick as shown with a force of 80 N, please find the approximate acceleration of the system of masses, and the tension in the string holding the 1 kg mass. In this problem, we use a dynamics lens and recognize that the acceleration is upward (or downward), so we choose x-y coordinates. a_x=0, so we know the normal force = the x component of the force I put on the stick.

A 9)

$\vec{a}_{sys} = ?$

$T = ?$



a) I use a dynamics lens since I see forces causing acceleration. $\Rightarrow \Sigma F = m\vec{a}$

$$\vec{F}_y - F_g - F_f = (m_1 + m_2)\vec{a}_{sys}$$

$$\vec{a}_{sys} = \frac{F_y - (m_1+m_2)g - \mu F_x}{m_1 + m_2} \qquad \mu = 0.4$$

$$\vec{a}_{sys} = \frac{70N - (3kg)(10 m/s^2) - (0.4)(39N)}{3kg}$$

$$\boxed{\vec{a}_{sys} \approx 8.2 \ m/s^2}$$

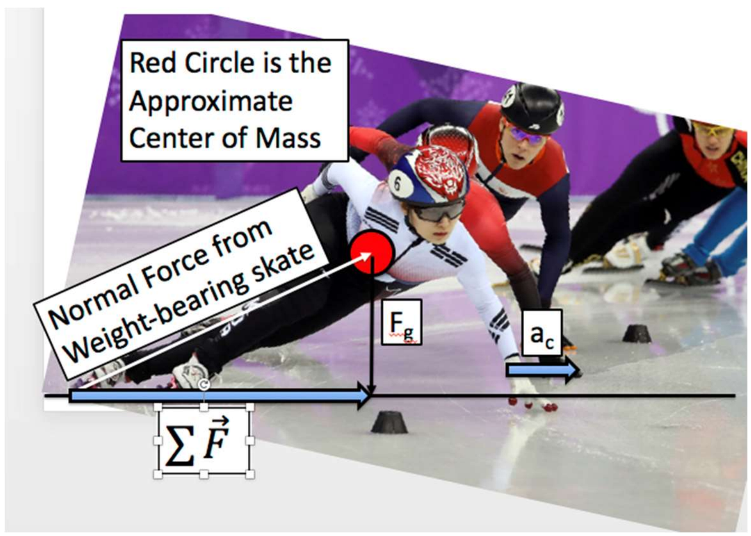
b) I use a dynamics lens for the same reason:

$$T - F_{g_2} = m_2\vec{a}$$

$$T = m_2\vec{a} + m_2 g$$

$$T = (1kg)(8.2 m/s^2 + 10 m/s^2) \approx \boxed{18.2N}$$

We're glad to see that the tension in the string > F_g because the mass attached to the string is accelerating upwards.



8. The 60 kg speed skater at right is executing a turn.
   a) If she is standing on one leg at this moment, estimate the force on her leg. Is this a lot of force? Could you stand on one leg with this much force on your leg?
   A dynamics lens is needed for this because the forces acting on this skater cause the (centripetal) acceleration. We write $\sum \vec{F} = m\vec{a}$, and make a free body diagram… clearly noting that she is not in equilibrium because she's moving in a circle, so we label the direction of the centripetal acceleration (into the center of the circle). We make sure then that we add the normal force from her leg and the force of gravity such that the $\sum \vec{F}$ is in the same direction as the acceleration. We see that the normal force is about 2.5 times as large as the force of gravity, so if her mass is 60 kg, the force of gravity on her is 600 N, and the normal force is about 1500 N. I think it would be very hard to hold 2.5 times your weight on one leg.
   b) The radius of curvature of these tracks is 15 m. Estimate the speed of this skater. Looking at the geometry above, we see that $\sum \vec{F}$ is about twice as large in magnitude as the force of gravity, or about 1200 N, yielding an acceleration of 2*g = 20 m/s^2… wow. This is her centripetal acceleration = v^2/R. Solving for v, we get $\sqrt{aR}$ =(300 m^2/s^2)$^{.5}$ ~ 17 m/s… like 38 mph, that's pretty fast no?

9. Please do your assessment #8 in fine fashion. Please see Assessment Solutions.