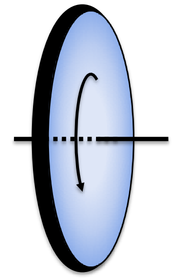
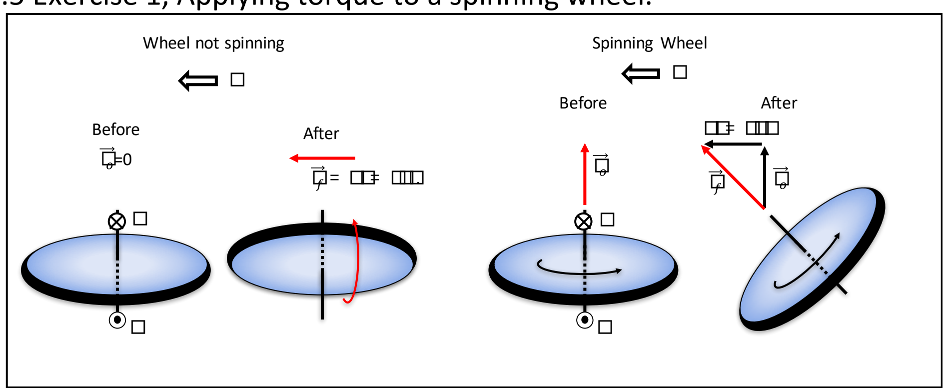
PS#10 Due in Class Monday, Dec. 3. Please pay good attention to describe the lens you are using and explain your method.

1. You are holding the axle of a bicycle wheel (one hand on each side) out in front of you, spinning as shown.

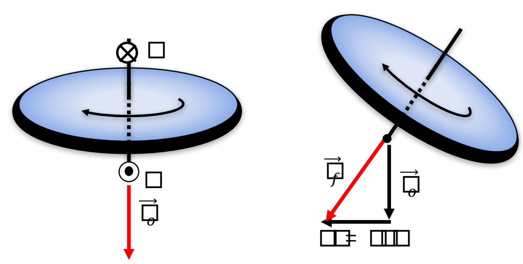


   a) What is the direction of the angular momentum vector?
      Right Hand Rule: to the right →
   b) You push away with your right hand and pull in with your left hand. What is the direction of the torque you put on the wheel? What is the direction of the angular impulse that you give to the wheel?
      Again, using the right hand rule, this is an upward torque, inducing rotation in the upward direction.
   c) After you push for a moment, how does the orientation of the wheel change?
      The upward torque provides some change in angular momentum in the upward direction or, $d\vec{l} = \vec{\tau}dt$. You add this $d\vec{l}$ to the wheel's previous angular momentum and find the new (resultant) angular momentum. This is still to the right, but slightly upward. Hence, you have rotated the angular momentum vector (and rotated the spinning wheel) counter clockwise, or (by the right hand rule) out of the paper at you.

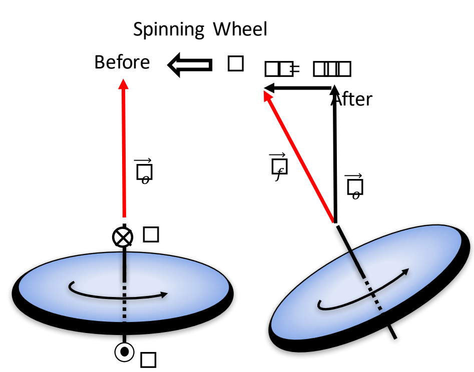
2. 7.5 Exercise 1, Applying torque to a spinning wheel.



- Can you predict the direction the axle will turn? You really should do this with a wheel and we did it.
- Predict what happens when the wheel is spinning in the opposite direction? Why?



If the wheel is spinning the opposite direction, the torque and change in angular momentum are the same, but the initial angular momentum is in the opposite direction. Thus, we see that the wheel will tip in the opposite direction. Thus it will precess in the opposite direction.



- What happens if the wheel is spinning faster? Can you explain why?

If the initial angular momentum is greater, then change of angular momentum from the same torque of gravity will result in a smaller change of angle of the spinning wheel. So it would precess slower.

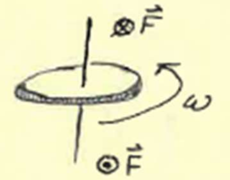
- How does the rate of precession change when you push harder on the axle? Why?

If you push harder on the wheel the torque and change in angular momentum will be greater. In the same amount of time, the wheel will tilt by more. Thus, the rate of precession will increase.

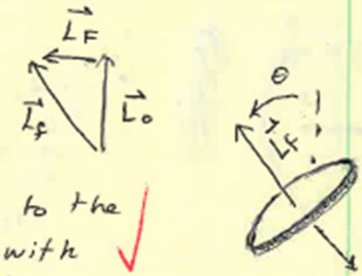
2 7.5 Ex. 1

Cons | Angular Momentum: $\Sigma\vec{L} = \vec{L}_o + \Delta\vec{L}$

Dynamics

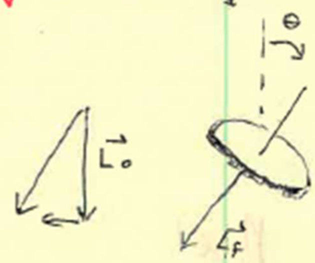


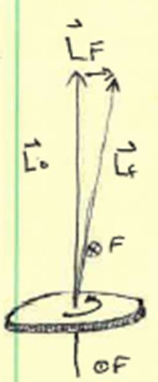
- The forces provide a ~~rightward~~ leftward angular momentum $\vec{L}$.



→ The axle will turn to the left from the top with the forces provided. ✓

→ Opposite direction: The $\vec{L}_o$ is downward, ~~So~~ the forces provide $\vec{L}$ to the left, so... ✓ The axle turns to the right from top ~~The axle will precess~~





* less direction change

→ If the wheel is spinning faster, there's more angular momentum up or down, so there's a greater vector magnitude, which is more difficult to change, & Axle direction will shift less. ✓

→ Rate of precession: Will increase with a larger force since a larger angular momentum ⊥ to the wheel is delivered, so it will ~~more~~ precess quicker. ✓

3. 7.5 Exercise 3 These questions are similar to those in the above questions. However, there are two exceptions:

- What changes if you switch sides and support the axle on the other side? Why?

Supporting the wheel from the other side will reverse the torque that gravity provides. This will result in the wheel precessing in the opposite direction. Please prove this to yourself with a drawing.

- What happens if you support the axle closer to the center of the wheel? Why?

Supporting the wheel close to the axle will reduce the torque from gravity. This will reduce the change in angular momentum, so the rate of precession will decrease.

Angular Momentum / Rotational Dynamics ✓

$\Delta\vec{L}$ is 0 so $\sum \vec{L}_{Total} = \vec{L}_o + \Delta\vec{L}$

• Forces applied at a radius create torques, (impulse on angular momentum)

$\vec{\tau} = \frac{d\vec{L}}{dt}$

• Supporting it on the ~~left~~ right side creates a torque and $\vec{L}$ into the paper. $\vec{L}_o$ is to the left. Summing these like vectors shows it will spin direction upward. ✓

• Spinning the wheel the opposite way creates a $\vec{L}_o$ to the right. The $\vec{\tau}$ forward sums with the downward $\vec{L}_o$, so spins downward. ✓

Nice!

• Faster spin = slower precession since there is a larger $\vec{L}_o$, so $\Delta\vec{L}$ will be more minimal compared to the large $\vec{L}_o$ ✓

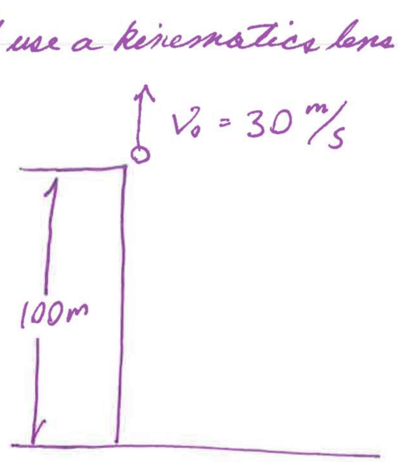
• Switching sides of the supporting axle, it will change the direction of $\vec{\tau}$ applied ~~so it will precess opposite~~ so it will precess opposite ✓

• Closer to the center, the torque applied by gravity will be less since $\tau = F_\perp(r)$, so a smaller r = smaller τ. This means slower precession. ✓

4. 7.6 Exercises 1 and 2, deriving our two kinematic equations. These are covered in the videos, and you don't have to hand them in, but it's a good exercise to do them in order to know where the formulas come from.

5. 7.6 Exercise 3, Throwing a rock upwards off the edge of a cliff.

I use a kinematics lens because we have motion: an explicit $f(t)$



$$y(t) = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$0 = \underbrace{100m}_{c} + \underbrace{30\,m/s}_{b}\, t \underbrace{- 5\,m/s^2}_{a}\, t^2$$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-30\,m/s \pm \sqrt{(30\,m/s)^2 - 4(-5\,m/s^2)(100m)}}{2(-5\,m/s^2)}$$

multiply through by (-1)

$$= \frac{30\,m/s \mp \sqrt{900\,m^2/s^2 + 2000\,m^2/s^2}}{10\,m/s^2}$$

$$= 3s \pm \sqrt{29}\ s$$

$$\approx 3s \pm 5.4s = -2.4s,\ 8.4s$$

The negative value.. given this trajectory, if we went backwards in time, it would be at the bottom of the cliff, moving upwards at about 54 m/s ... $= 30\,m/s + g(2.4s)$

But we didn't need the quadradic equation. We knew all along how to find time:

$\Delta X = V_{ave}\,\Delta t$ $\quad \Delta t = \dfrac{\Delta X}{V_{ave}}$ $\quad V_{ave} = \dfrac{(V_0 + V_f)}{2}$

we can use this given constant acceleration (g)

We can find V_f using an energy lens because

$E_p \Rightarrow E_K \qquad E_o = E_f$

$E_K + E_p = E_K$

$mgh_o + \frac{1}{2}mv_o^2 = \frac{1}{2}mv_f^2$

$v_f = (v_o^2 + 2gh_o)^{\frac{1}{2}}$

$= \left[(30\,m/s)^2 + 2(10\,m/s^2)100\,m\right]^{\frac{1}{2}}$

$\approx 54\,m/s \qquad + \uparrow$

$V_{ave} = \frac{(30\,m/s + -54\,m/s)}{2} \approx -12\,m/s \qquad \Delta x = -100\,m$

$\Delta t = \frac{\Delta x}{V_{ave}} = \frac{-100\,m}{-12\,m/s} \approx \underline{\underline{8.3\,s}}$ ✓

6. 7.6 Exercise 4, Catching the Bus.

PS #6

#1 - Catching Bus

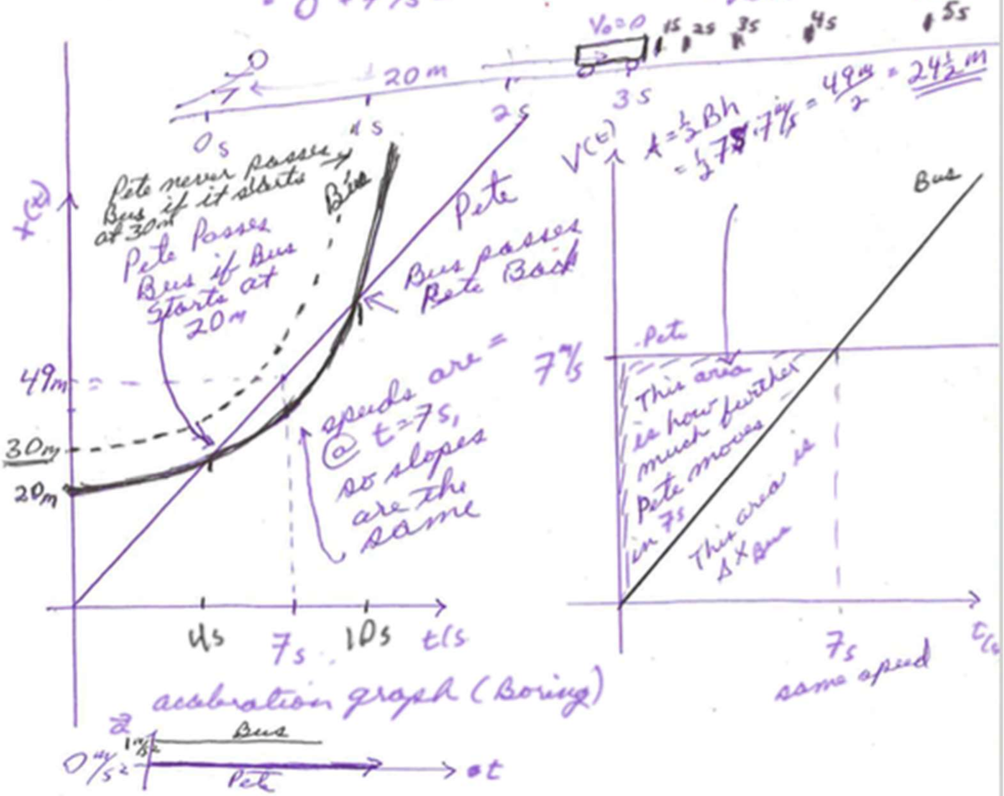
Pete: $V_P = 7\,\text{m/s} = \text{const}$, $x_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,\text{m/s}^2$, $x_0 = 20\,\text{m}$, $V = a_B t$

Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $X_P(t) \stackrel{?}{=} X_B(t)$ when and if are our displacements the same

Pete: $X = X_0 + Vt$
$= 0 + 7\,\text{m/s}\; t$

Bus: $X = X_0 + V_0 t + \frac{1}{2} a t^2$
$= 20\,\text{m} + 0 + \frac{1}{2}\,\text{m/s}^2\, t^2$



6 7.6 Ex. 4

$\vec{V} = 7\,m/s$

20m

$\vec{a} = 1\,m/s^2$

Lens: kinematics, we have velocity and $\vec{a}$ as a fn of time.

~~$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$~~

~~$= 20m + 7\,m/s\, t + \ldots$~~

Person

$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$

$x(t) = 0 + 7\,m/s\, t + 0$

$x_p(t) = 7\,m/s\, t$

Bus

$x_B(t) = 20m + 0 + \frac{1}{2}(1\,m/s^2)t^2$

$x_B(t) = 20m + \frac{1}{2}\,m/s^2\, t^2$

$$7\,m/s\, t = 20m + \frac{1}{2}\,m/s^2\, t^2$$

$$0 = \frac{1}{2}\,m/s^2\, t^2 - 7\,m/s\, t + 20m$$

$$t = \frac{7 \pm \sqrt{49 - 4(\frac{1}{2})(20)}}{1} = 7 \pm \sqrt{9}$$

$t = 7 \pm 3$

$t = 4, 10$

$x_p(t) = 7\,m/s\, t$

$x_p(4) = 7\,m/s\,(4s) = \boxed{28\,m}$ @ 4s

$x_p(10) = 7\,m/s\,(10s) = \boxed{70\,m}$ @ 10s

You catch the bus

7. 7.6 Exercises 5 – 7 (Pulling sled, Hitting a baseball, Torque on a wheel.

7.6 Exercise 5. We would solve this problem exactly as we did before we used trigonometry. The only difference is now we could calculate the components rather than just eyeball (estimate) them. Of course, we recognize this as a dynamics problem whereby the acceleration is horizontal, so we choose x-y components and break the tension into horizontal and vertical components.

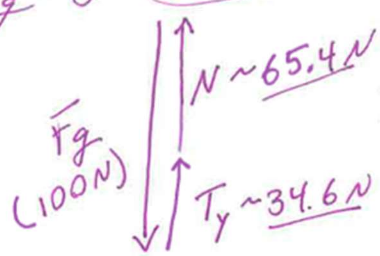
$T_x = T\cos(30^o) \sim 40\ N*(0.866) = 34.6\ N$

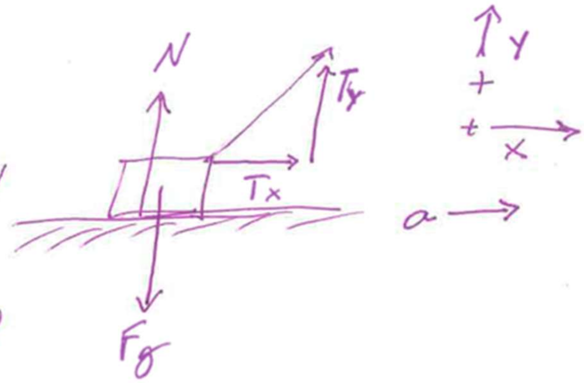
$T_x = T\sin(30^o) \sim 40\ N*(0.5) = 20\ N$.

$W = \vec{F} \cdot \overrightarrow{dx}$, We take the x-component of the tension (force) to find that the work I do is *20 N * 5 m = 100 J.*

$\Sigma F_y = 0$

$T_y + N - F_g = 0$ $\quad \Sigma F_y = 0$





$\Sigma F_x = ma$

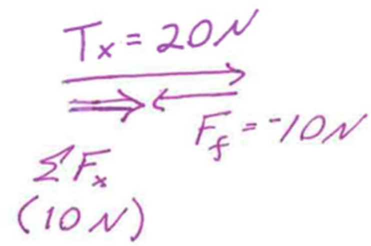
$T_x - F_f = ma$

$20N - 10N = ma$

$\frac{10N}{m} = \underline{\underline{a \approx 1\,m/s^2}} \Rightarrow$

$F_f = \mu N$

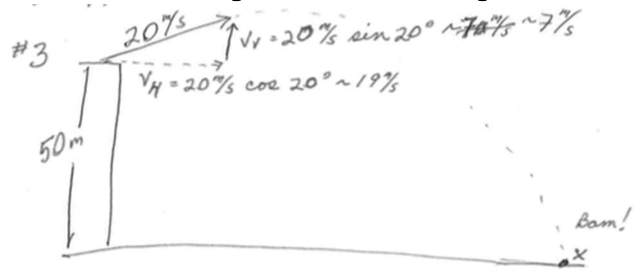
$= 0.15 \cdot 65.4N \sim 10N$



finding the acceleration requires us to use a dynamics lens because the force cause the acceleration. We do a good FBD as always and identify that the forces in the x direction are the horizontal tension and the friction force. To find the force of friction, we need the normal force. We recognize that we are in equilibrium in the y direction because we are (likely) not accelerating off the surface of the earth. Gravity provides 100 N of force (downward), and the vertical component of tension is 34.6 N upward. In order to be in equilibrium in the y direction, the normal force must be 65.4 N (upward). This yields a friction force of about 10 N in the direction opposite to our motion. Assuming that we are moving forward as I pull the sled, the net force is the sum of the x-component of tension minus the frictional force *20 N – 10 N = 10 N* in the positive direction. This yields an acceleration of the 10 kg sled and girl of 1 m/s^2.

7.6 Exercise 6: If you hit a baseball at a 20 degree angle above the horizon, at an initial velocity of 20 m/s off the edge of a cliff 50 m high,

#3

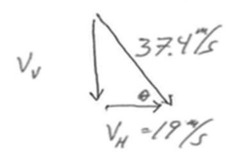


a) $E_o = E_f$

$KE_o + PE_o = KE_f + \cancel{PE_f}^{\,0}$ solve ~~for~~ math and find $V_f \approx 37.4\,m/s$

$\frac{1}{2}\cancel{m}(20\,m/s)^2 + \cancel{m}\,10\,m/s^2 \cdot 50\,m = \frac{1}{2}\cancel{m}V_f^2$

b) $a_H = 0$, so $V_{H_{final}} = V_{H_{initial}} = \text{const} \approx 19\,m/s$



$\cos\theta \approx \frac{19\,m/s}{37.4\,m/s} \approx 60°$

$V_V^2 + V_H^2 \approx V^2$

$V_V^2 + (19\,m/s)^2 = (37.4\,m/s)^2$

$V_V \approx 32\,m/s \Downarrow$

c) $\Delta V_V = -32\,m/s - 7\,m/s$

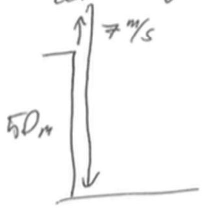
$\approx -40\,m/s = at$

$a \approx -10\,m/s^2$, so $\Delta t \sim 4\,s$

$\Delta x = V_H t + \cancel{\tfrac{1}{2}a_H t^2}^{\,0} \approx 19\,m/s \cdot 4\,s \approx \underline{\underline{75\,m}}$

horizontal $\vec{a} = 0$

d) now I use straight kinematics.
Horizontally, it's just moving along at $V_H \approx 19\,m/s$ in the $\hat{x}$ direction.
vertically it's moving upward at $V_{V_0} \sim 7\,m/s$ and accelerates downward at $a_V = -10\,m/s^2$



find t w/

$X_f = 0 = 50\,m + 7\,m/s + \frac{1}{2}(-10\,m/s^2)t^2$

solve!

$t \approx 4\,s$ (and $\sim -2.5\,s$, but that's not relevant)

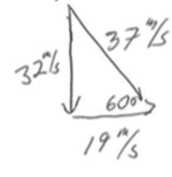
Then you can find $X_h = V_H t$ and

$V_V = V_{0V} + at$

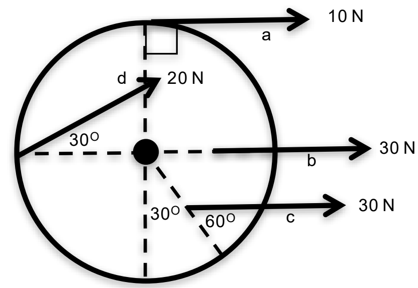
to get same answers as before!

e) $X_f \approx 75\,m$

f



a. Exercise 7: Using some geometry, please show that I have correctly labled the algne to be 60°, that it is the complement of the 30° central angle. We know $\vec{\tau} = F_{\perp} r = F r_{\perp} = F r(\sin\theta_{included})$. THUS!



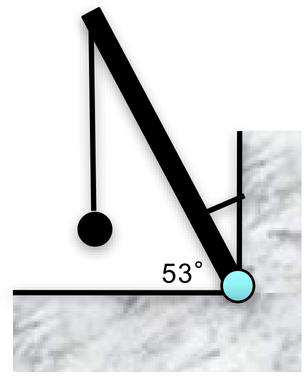
$a)\ \vec{\tau} = 15\ Nm \otimes$

$b)\ \vec{\tau} = 0$

c) $\vec{\tau} = 0.75\ m * 30\ N(\sin 60^o) = 19.5\ Nm \odot$

d) $\vec{\tau} = 1.5\ m * 20\ N(\sin 30^o) = 15\ Nm \otimes$

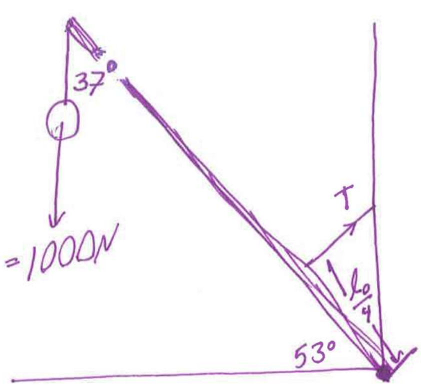
8. In the diagram at right, a post of some length supports a 100 kg ball. The length of the tilted rod is 10 m and the cable is attached 2.5 m from the pivot. From the drawing at right (make your own better drawing), estimate the tension on the cable and the force provided by the foundation at the pivot.



I assign directions

$\longrightarrow +x$ $\uparrow +y$

$\otimes$ + rotation



There are unknown forces at the pivot so I use the pivot as the center of rotation, leaving the only unknown torque that of the Tension:

(length of rod)

$\Sigma \tau_{Pivot} = {}^{+}T \cdot \frac{l_0}{4} \sin 90^\circ + {}^{-}F_g \cdot l_0 \sin 37^\circ = 0$

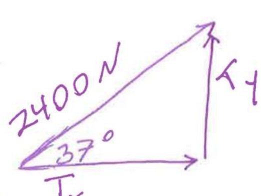
l_0 cancels and $\sin 37^\circ \sim \frac{3}{5} = 0.6$

$T \sim 4 \cdot F_g \cdot \sin 37^\circ \sim 4 \cdot 1000N \cdot 0.6 = \underline{2400\,N}$

In order to find the reaction force provided by the pivot, we set $\Sigma F = ma = 0$. I choose to decompose the Tension into T_x and T_y

$T_y = 2400N \sin 37^\circ \sim 1440\,N$

$T_x = 2400N \cos 37^\circ \sim 1920\,N$



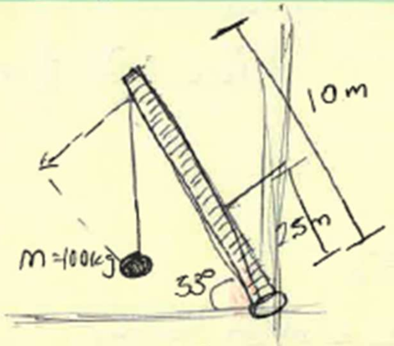
$\Sigma F_x = F_{px} + T_x = 0;\ F_{px} = -1920N\,\hat{x}$

$\Sigma F_y = F_{py} + T_y + F_g = 0;\ F_{py} = -440N\,\hat{y}$

We see that the reaction force that the pivot provides is downward and to the left.

Please show yourself that if the cable was connected in the *middle* of the supporting rod, the tension on the cable would be only 1200 N, resulting in a pivot force that would be 280 N upward and 960 N in the negative x direction.

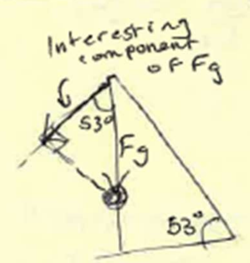
8



Rotational Dynamics : STATICS

$\Sigma F = 0 \qquad \Sigma \tau = 0$

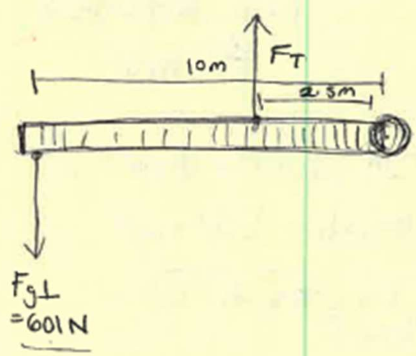
Let's choose

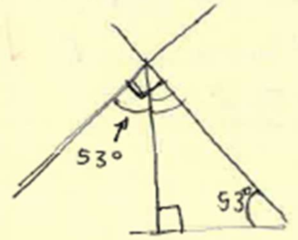


$\cos 53° = \dfrac{F_{g\perp}}{F_g}$

$0.601\,(1000\,N)$

$= 601\,N$





~~$\tau_g = 6010\,N\cdot m$~~

~~$\tau_T = 601N(2.5m)$~~

$\Sigma\vec{\tau} = (601\,N)(10m) + F_T(2.5$

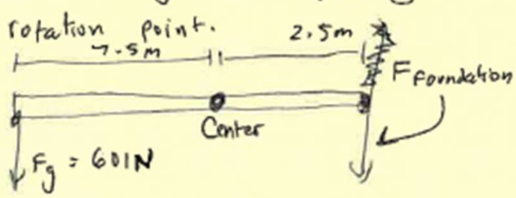
$0 = 6010\,N\cdot m + (-F_T)2.5m$

$\dfrac{6010\,N\cdot m}{2.5m} = F_T$

$\boxed{F_T = 2404\,N}$

TENSION

For finding foundation, change rotation point.



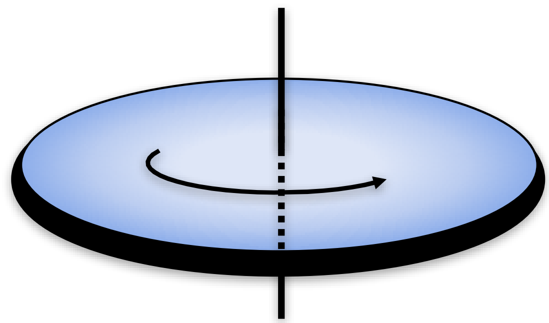
$\Sigma\vec{\tau} = \overset{F_g(r)}{601N(7.5m)} + \overset{F_{foundation}(r)}{(+F_{foundation})(2.5m)}$

$0 = \dfrac{4507.5\,N\cdot m}{2.5} = F_{foundation}$

$\boxed{F_{foundation} = 1803\,N}$

Great!

9. At right is shown a spinning solid disk on a vertical axis 80 cm long (40 cm above and 40 cm below) spinning in space in front of you, balanced on its axes. It has a mass of 2 kg and a radius of 0.5 m and is spinning around 10 times per second!



a) Show that the angular momentum of this disk is about 15 Js, and define the direction of the angular momentum vector.

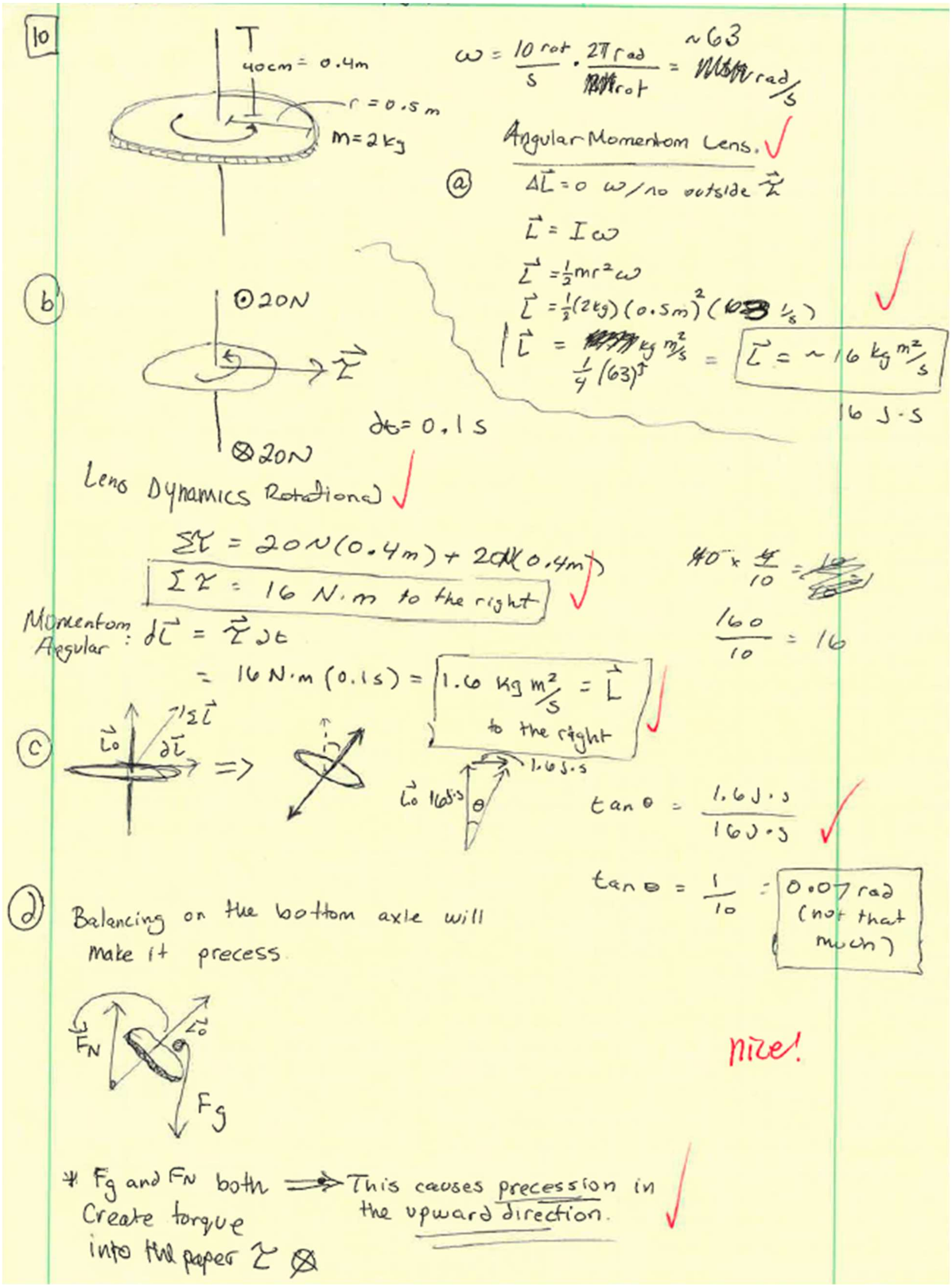
b) Then, looking at the paper, you grab the top of the axel with one hand and the bottom of the axel with the other hand. You pull the top out toward you with 20 N and push the bottom away from you with 20 N of force. You do this for a tenth of a second (0.1 s). Calculate the torque you apply to the wheel (include direction), and the angular momentum you impart onto the wheel. Remember to include direction.

c) After you do the above act for 0.1 s, you let the wheel go again. Is the wheel tilted now? If not, why? If so, in which direction and by about how much? Draw the wheel in its new position.

It tilts to the right, rotating in the direction into the paper, by about 1/10 of a radian… ~ 10 degrees.

d) What will the wheel do now if I let it balance on the bottom of the axel? Describe the motion as exactly as possible. Now that it's tipping, gravity will provide a torque into the paper, so that will make the top precess around in the upward direction (counter clockwise when viewed from above).

See BELOW



10

T

40cm = 0.4m

r = 0.5 m

m = 2 kg

$\omega = \frac{10\ \text{rot}}{s} \cdot \frac{2\pi\ \text{rad}}{\text{rot}} = \sim 63\ \text{rad}/s$

Angular Momentum Lens. ✓

(a) $\Delta \vec{L} = 0$ w/ no outside $\vec{\tau}$

$\vec{L} = I\omega$

$\vec{L} = \frac{1}{2} m r^2 \omega$

$\vec{L} = \frac{1}{2}(2\text{kg})(0.5\text{m})^2(63\ 1/s)$

$|\vec{L}| = \frac{1}{4}(63)^1\ \text{kg}\ m^2/s = \boxed{\vec{L} = \sim 16\ \text{kg}\ m^2/s}$

16 J·s

(b) ⊙ 20N

$\vec{\tau}$

⊗ 20N

$dt = 0.1\ s$

Lens Dynamics Rotational ✓

$\Sigma\tau = 20N(0.4m) + 20N(0.4m)$

$\boxed{\Sigma\tau = 16\ N\cdot m \text{ to the right}}$ ✓

$40 \times \frac{4}{10} =$

$\frac{160}{10} = 16$

Momentum Angular: $d\vec{L} = \vec{\tau}\, dt$

$= 16\ N\cdot m\,(0.1s) = \boxed{1.6\ \text{kg}\ m^2/s = \vec{L} \text{ to the right}}$ ✓

(c) $\vec{L}_0$ $\Sigma\vec{L}$ $d\vec{L}$ =>

$\vec{L}_0$ 16 J·s θ 1.6 J·s

$\tan\theta = \frac{1.6\ J\cdot s}{16\ J\cdot s}$ ✓

$\tan\theta = \frac{1}{10} = \boxed{0.07\ \text{rad (not that much)}}$

(d) Balancing on the bottom axle will make it precess

F_N $\vec{L}_0$ F_g

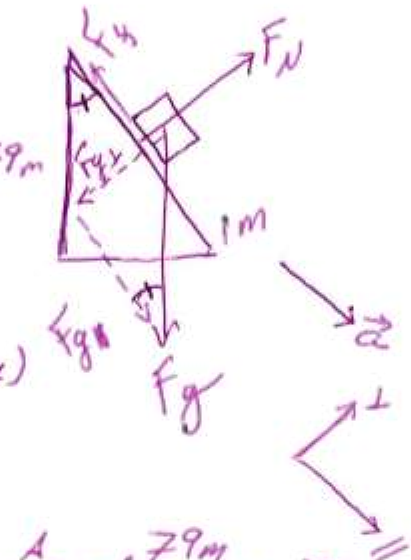
* F_g and F_N both Create torque into the paper τ ⊗ ⟹ This causes precession in the upward direction. ✓

nice!

10. Do Assessment #9 in fine form. Please see assessment #9 solutions
11. We calculated the coefficient of friction between the calculator and the metal wheel two different ways:

a) The angle that the calculator slid at was when the opposite side was 79 cm and the hypotenuse was 1 m.
b) The table was 108 cm high, the radius was 18 cm, and the horizontal distance was 90 cm. When it fell onto the table, it fell 17 cm and moved a distance of 42 cm

11. This is a dynamics lens because $\vec{F} \Rightarrow a$, more Precisely:



$\Sigma \vec{F} = m\vec{a} = 0$ (immediately before it slips)

$\Sigma F_{\perp} = ma_{\perp} = 0$

$F_N + {}^{-}F_{g\perp} = 0$

$F_{g\perp} = F_N = F_g \cdot \cancel{\cos\theta}\ \sin\theta$
$= mg\,\cancel{(.79)}(.61)$

$\cos\theta = \frac{A}{H} = \frac{.79m}{1m} = .79$

$\cos^2\theta + \sin^2\theta = 1$

$\sin\theta = (1-(.79)^2)^{\frac{1}{2}}$
$= 0.61$

or $\theta \approx 37°$
base $\angle \approx 53°$

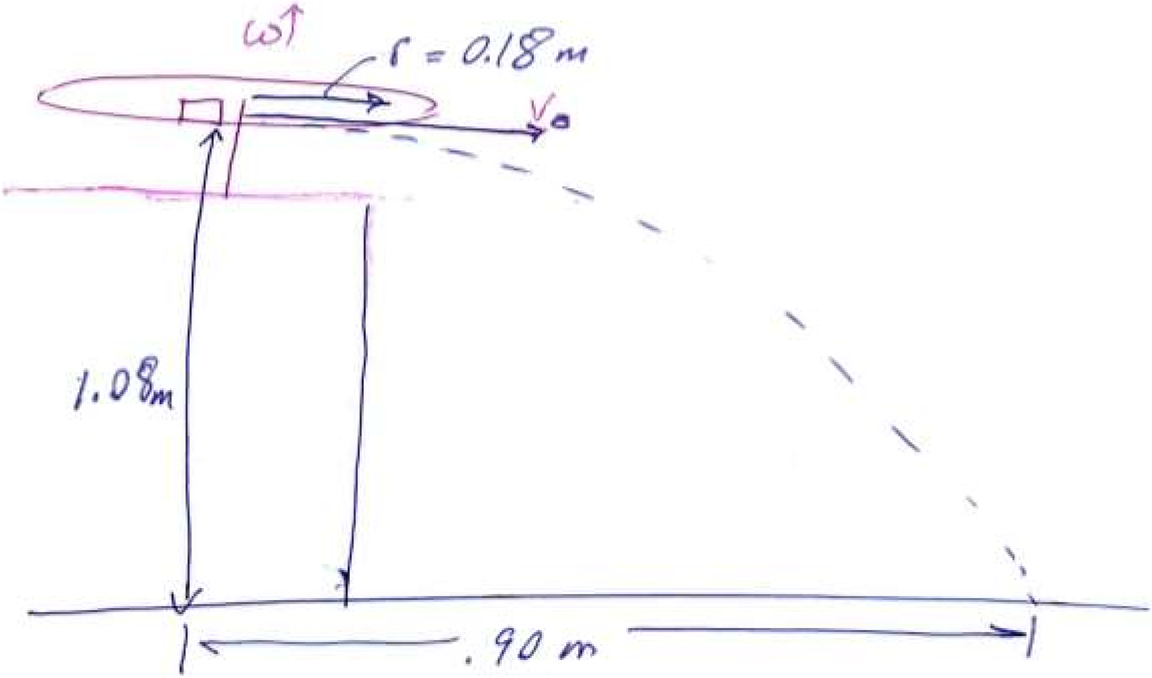
$\Sigma F_{\parallel} = ma_{\parallel} = 0$

$F_{g\parallel} + {}^{-}F_f = 0$

$mg\,\cancel{\cos}\ \sin\theta = +F_f = \mu \cos\theta\, mg$

$\mu = \frac{\cancel{mg}\cos\theta}{\cancel{mg}\sin\theta} = \frac{1}{\tan\theta} = \frac{.79}{.61} \approx 1.3$

most of you probably defined the base $\angle$ as θ, so you have the opposite $\sin\theta \leftrightarrow \cos\theta$, but should get the same answer



I know this is a dynamics lens because

$F_f \Rightarrow a_c$ or $F_f = ma_c$

$\mu F_N = m\frac{V^2}{r}$ ... but I need $V = V_H$

$\mu mg = \frac{mv^2}{r}$

Kinematics lens, because we know ~~...~~ explicit motion $a_x = 0$ because in the air, the only force is $F_g \Downarrow$ $V_H = V_{ave} = \frac{\Delta x}{\Delta t}$.

Δt is determined by the ~~fally~~ falling time.

$\Delta y = \frac{1}{2}gt^2$ because $V_y = 0$ at $t = 0$

$t = \left(\frac{2\Delta Y}{g}\right)^{\frac{1}{2}} = \left(\frac{2 \cdot 1.08\,m}{10\,m/s^2}\right)^{\frac{1}{2}} = 0.47\,(s^2)^{\frac{1}{2}} = 0.47\,s$

$V_H = \frac{\Delta x}{\Delta t} = \frac{.90m}{.47s} = 1.92\,m/s$ $\quad a_c = \frac{V^2}{r} = \frac{(1.92\,m/s)^2}{.18m} \approx 20\,m/s^2$

$\rightarrow \mu = a_c/g \approx 2$

So, why didn't the two different calculations match? There were different errors that could have happened during the experiments. In particular, I'm pretty sure that between the "ramp sliding" experiment and the measurement of the angle, we let the meter stick drop a little. I think that the actual angle was higher… there is a need to be more careful next time! But friction is a tricky thing. For instance if we hiccup during the experiment, the experiment vibrates and can break the static friction into the (lower) dynamic friction.