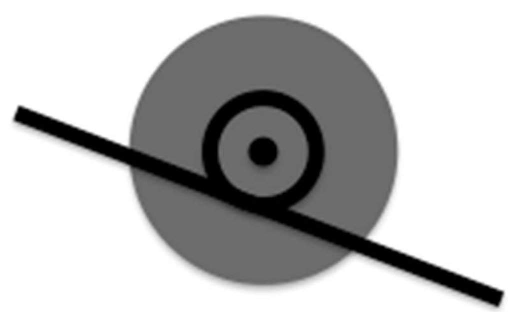
PS#10 Due Thursday, March 15 in class. Remember to start each question with a description of the lens and method.

1) A bicycle is a beautiful thing to me! This question is largely addressed through the "bicycle transmission" video in Week 10. Please see the video again if you are so inclined. However, I will address right here question H) What happens when you change the rear gear to twice the original radius? This is downshifting by a factor of two. At that moment, there is no immediate change of speed, so the rear wheel has the same rotational velocity. With twice the radius on the rear cog (gear), the chain must move twice the speed to keep up with the rotating wheel. Because the chain has the same tension on it (you are pushing with the same force on your feet), the power supplied by the chain ($P = F*v$) is doubled. Because the chain tension acts on the rear wheel at twice the radius, the chain's torque on the rear wheel doubles, doubling the force to the earth's surface (and the force of the earth's surface on the bike doubles). By doubling the force to the rear wheel, the bike will accelerate, and you've doubled the power delivered to the rear wheel. In order to move the chain twice as fast, you will need to spin your legs twice as fast, doubling your power output if you are able to continue pushing with the same force on your legs: $P = \tau\omega = Fv$. This is what we experience every day… if you are cruising at constant velocity on the freeway, your engine is not putting out very much power. But then you want to pass someone. You downshift to a lower gear (or your automatic transmission does that). The engine spins much faster, which you can hear. The power to the wheels increases greatly and you accelerate increasing the kinetic energy of your car. Same thing on a bicycle. Now can you answer the questions:

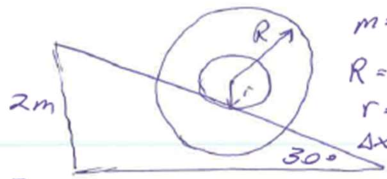
2) Remember the flywheel from a previous problem set?



a) $\Delta h = 2m$

$\Delta PE = mg\Delta h = 3kg \, 10 \, m/s^2 \, 2m$

$= 60 \frac{kg\,m^2}{s^2} = 60J$




$m = 3kg$
$R = .3m$
$r = .1m$
$\Delta x = 4m$

b) Energy conservation –

$I = \frac{1}{2} MR^2$

$V = \omega r$

$PE \Rightarrow KE_{linear} + KE_{rotation}$

$= \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} m\omega^2 r^2 + \frac{1}{2}\,\frac{1}{2} mR^2\omega^2$

$\cancel{m} g\Delta h = \frac{1}{2}\cancel{m}\omega^2 (r^2 + \frac{1}{2}R^2) = \frac{1}{2}\cancel{m}\omega^2 \underbrace{(10^{-2} m^2 + \frac{1}{2}.09m^2)}_{0.55\ m^2}$

$\omega^2 = \frac{2}{0.055\,m^2} \, 10 \, m/s^2 \, 2m \approx 727/s^2$

$\boxed{\omega_f \approx 27/s}$

$\omega_{ave} = \frac{\omega_f}{2} \approx 13.5/s$

$V_{ave} = \omega_{ave} \cdot r = 1.35\, m/s = \frac{\Delta x}{t}$

$t = \frac{\Delta x}{V_{ave}} = \frac{4m}{1.35\, m/s} \approx 3.0s$

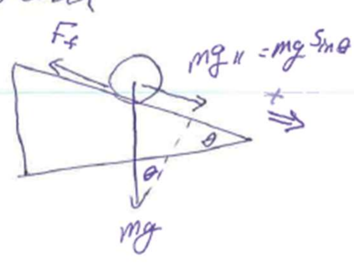
$\alpha = \frac{\Delta\omega}{\Delta t} \approx 9.1/s^2 \qquad a = \alpha r = 0.91\, m/s^2$

$\tau = \alpha I = \frac{9.1}{s^2} \frac{1}{2} mR^2 = \frac{9.1}{s^2} (\frac{1}{2}) . 3kg\,(.3m)^2 = 1.23\, Nm$

$F_f = ?\quad \tau = F_f r \quad F_f = \frac{\tau}{r} = 12.3N$ (up the hill)

This is now a dynamics problem

$\Sigma\vec{F} = m\vec{a} = mg_{\parallel} - F_f$

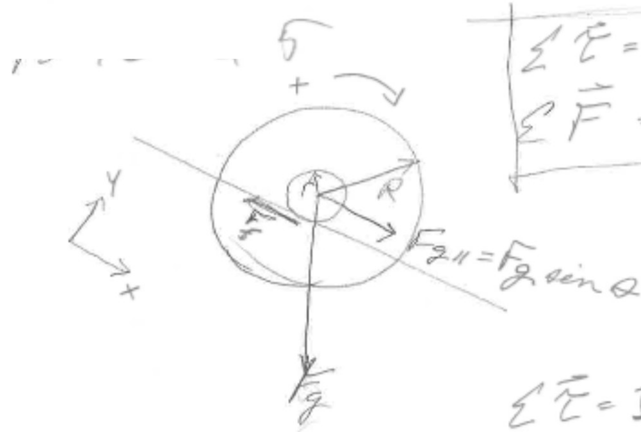



$= 15N - 12.3N \approx 2.7N$

$a = \frac{\Sigma\vec{F}}{m} = \frac{2.7N}{3kg} \approx 0.9\, m/s^2$

which is what we calculated previously.

Below is a different way to find the answers using a dynamics approach and solving the simultaneous equations. ***BUT at the very end, I show you how to solve it in just one line*** by just saying that at this instant in time, the wheel is pivoting ***around the point of contact*** and finding the torque = $F_{g(parallel)}$*r, and using the parallel axis theorem to find the moment of inertia of the wheel about this point.




$$\boxed{\begin{array}{ll}\sum \vec{\tau} = I\vec{\alpha} & a = \alpha r \\ \sum \vec{F} = m\vec{a} & \end{array}}$$

$\sum \vec{\tau} = I\vec{\alpha}$ $\quad$ $\sum F = ma$

$F_f r = I\alpha$ $\quad$ $F_{g\parallel} - F_f = ma$

$F_{g\parallel} - ma = F_f$

$F_{g\parallel} r - m\alpha r^2 = I\alpha$ $\quad$ $F_{g\parallel} - m\alpha r = F_f$

$F_{g\parallel} r = I\alpha + mr^2\alpha$

$$\boxed{\frac{F_{g\parallel} r}{\underbrace{I + mr^2}_{I_{PA}}} = \alpha}$$

$$\alpha = \frac{(\cancel{m} g \sin 30^\circ)(0.1\,m)}{\frac{1}{2}\cancel{m}(0.3m)^2 + \cancel{m}(0.1m)^2}$$ mass cancels

$$= \frac{5\,m/s^2\ (.1m)}{.055\ m^2}$$

$$= 9.1/s^2$$

$$a = \alpha r = 0.91\ m/s^2$$

$$\tau_{Pt\ of\ contact} = I_{Parallel\ axis}\ \alpha$$

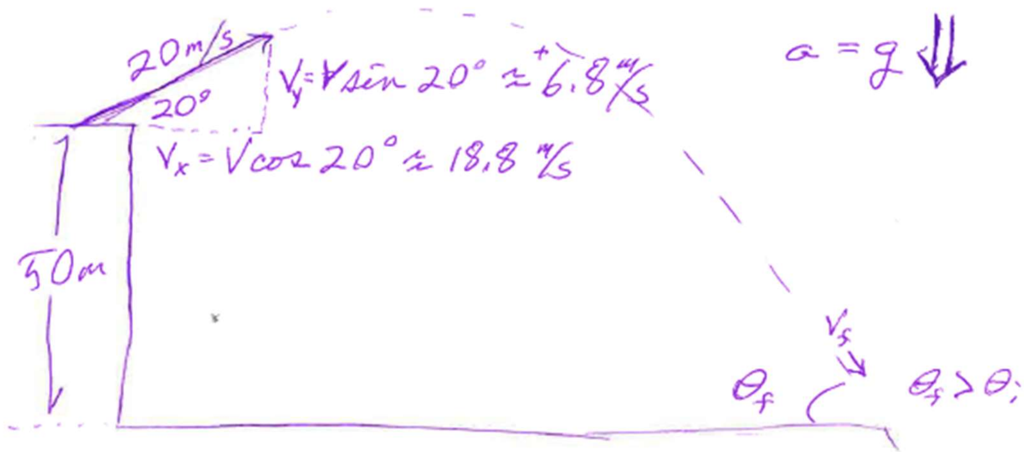
$F_{g\parallel} \cdot r = I_{PA}\,\alpha$ $\quad$ $\alpha = \frac{F_{g\parallel} \cdot r}{I_{PA}}$

3) The classic “notorious ladder problem” Please see the dedicated video.

4) Calculating coefficient of friction.
   I solved this in the last problem set solutions, PS#10

5) Hit a baseball off a cliff: Exercise 6, section 7.6
   There’s two ways to solve this that I know if. Strictly kinematics, you can make a good drawing and decompose the initial velocity into vertical and horizontal components. We do this because of dynamics because (Gravitational) force cause acceleration (downward). It is the *time* that connects the vertical situation (the ball goes upward, stops, comes downward, with downward acceleration of gravity) while the in the horizontal direction, the ball moves forward at a constant horizontal speed until it hits the ground. It is TIME that connects the two – the ball only moves horizontal for the same amount of time that it is moving up and down. We solve the vertical (quadratic) equation for time, and substitute it into the horizontal equation for constant speed in the x direction to get the distance the ball goes forward before hitting the ground. Then we can look at the vertical velocity! We use $v_f = v_i + -gt$ to find the final vertical velocity and add this to the horizontal velocity in order to get the final velocity. We use trig to find the angle.

   But, I like energy! First I’d make a good drawing. I would use energy to solve this problem because $E_k + E_g \Rightarrow E_k$. Using this, I find $v_f$ then $v_{yf}$ the time, then angle, then distance.




I'll use an energy lens because

$$E_k + E_g \Rightarrow E_k$$

$$\tfrac{1}{2}mV_i^2 + mgh_i = \tfrac{1}{2}mV_f^2 + \cancel{mgh_f} \to 0$$

$$V_f^2 = V_i^2 + 2gh = (20\,m/s)^2 + 2(10\,m/s^2)50\,m$$

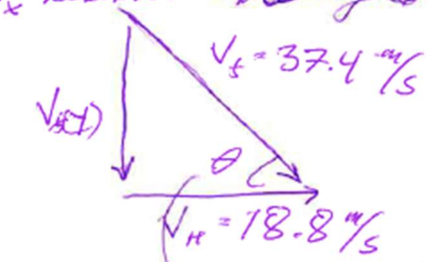
$$= 400\,m^2/s^2 + 1000\,m^2/s^2$$

$$= 1400\,m^2/s^2$$

$$V_f = (1400\,m^2/s^2)^{\frac{1}{2}} = 37.4\,m/s$$

reconstructing $V_f$, we know $V_x$ hasn't changed

$$V_H = V_x = 20\,m/s \cos 20° \approx 18.8\,m/s$$




using Pythagorus:

$$\cancel{V_{fy}} \quad V_{fy}^2 + V_H^2 = V_f^2$$

$$\theta = \tan^{-1}\left(\frac{32.4\,m/s}{18.8\,m/s}\right) = 59.9°$$

$$\text{or } V_{fy} = \cancel{41.9\,m/s}\ 32.4\,m/s = V_{iy} + {}^{-}gt,$$

$$a = \frac{\Delta V}{\Delta t}, \text{ so } \Delta t = \frac{\Delta V}{a} = \frac{V_f - V_i}{a} = \frac{-\cancel{41.9}\ 32.4\,m/s - 6.8\,m/s}{-10\,m/s^2}$$

$$= \sim 3.9\,s$$

So, the distance traved horizontally is:

$\Delta x = V_{ave} \cdot \Delta t = V_H \cdot \Delta t \approx 18.8\,m/s \cdot 4.9\,s \approx 91.5\,m$

Kinematics because we are looking for motion as an explicit function of time.

Using the same drawing, we separate the motion into x, y equations

x, horizontal $a = 0$

$x_f = \cancel{x_i} + V_i t + \frac{1}{2}\cancel{a}t^2$, $V_i = 18.8\,m/s$

$x_f = 18.8\,m/s \cdot 3.9\,s = 73.6\,m$

$\cancel{= V_i \Delta t}$

$V_f = V_i + \cancel{a}t = V_f = 18.8\,m/s$

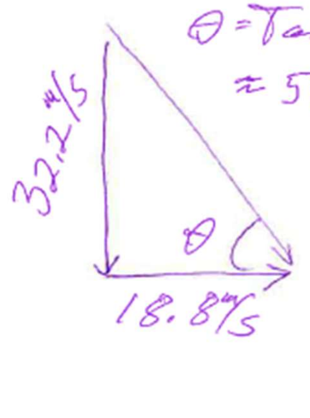
$=$

$V_f = ((32.2\,m/s)^2 + (18.8\,m/s)^2)^{\frac{1}{2}}$

$= 37.1\,m/s$

$\theta = \tan^{-1}\left(\frac{32.2}{18.8}\right)$

$\approx 59.7^\circ$




y, vertical $a = -g$

$y_f = y_i + V_i t + -\frac{1}{2}gt^2$ $V_i = 6.8\,m/s$

$0 = \underbrace{y_i - y_f}_{50\,m = c} + \underbrace{V_i}_{b} t \underbrace{- \frac{1}{2}g}_{a = -5\,m/s^2} t^2$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6.8\,m/s \pm ((6.8\,m/s)^2 - 4 \cdot (-5\,m/s^2)(50\,m))^{\frac{1}{2}}}{2(-5\,m/s^2)}$

$= .68\,s \pm 3.24\,s$, $-2.6\,s$, $3.9\,s$

$3.9\,s$

$V_f = V_i + at = 6.8\,m/s - 10\,m/s^2 (3.9\,s)$

$= 6.8\,m/s - 39\,m/s$

$\approx -32.2\,m/s$