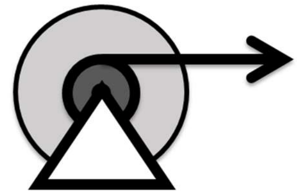
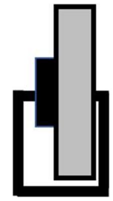
Assessment #6 121 Schwartz

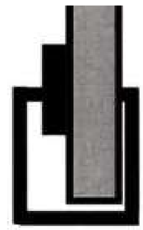
1. You pull on a string wrapped around the inner pulley on a round, flat disc flywheel as shown in the drawing. Starting from rest, the wheel accelerates at a rate of 10 s^{-2}. The disc is 4 kg with a radius of 2 m. The inner pulley has no mass and a radius of 80 cm.
   a) After pulling for 4 s, find the rotational velocity of the wheel.
   b) After 4 s, how far has the wheel rotated?
   c) How hard did I pull on the string?





Side View

c) How hard did I pull on the string?



Side View

I will use a rotational kinematics lens since I know the angular acceleration and I'm looking for the rotational velocity.

$\alpha = 10\ s^{-2}$

$m = 4\ kg$

$r = 2\ m$

a) $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t}$

$\Delta\vec{\omega} = \vec{\alpha}\cdot\Delta t$

$\Delta\vec{\omega} = 10\ s^{-2}\cdot 4s = \boxed{40\ s^{-1}}$

b) $\vec{\omega}_{ave} = \dfrac{\Delta\theta}{\Delta t}$

$\Delta\theta_{ave} = \vec{\omega}_{ave}\cdot\Delta t$

$\Delta\theta_{ave} = 40\ s^{-1}\cdot 4\ s = \boxed{160\ \text{radians}}$

Since rotational velocity is equal to the change in distance (radians) over time, I can use it to find how far the wheel has rotated.

c) I will use the rotational dynamics lens because torque causes not acceleration. It is a disc so its moment of inertia = $\frac{1}{2}mr^2$. And the perpendicular force from the string being pulled is directly related to torque.

$\tau = I\alpha$

$\tau = \left(\frac{1}{2}\cdot 4\ kg\cdot 2m^2\right)\left(10\ s^{-2}\right)$

$\tau = 80\ N\cdot m$

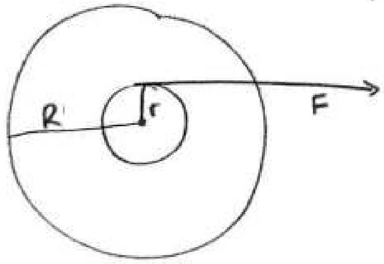
$\tau = F_\perp r$ 😊

$F_\perp = \dfrac{\tau}{r} = \dfrac{80\ N\cdot m}{\cancel{2\ m}\ 0.8\ m} = \cancel{40\ N} = 100\ N$

$\boxed{F_\perp = \cancel{40\ N}}$ $\boxed{F\ 100\ N}$

A+

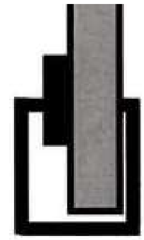
c) How hard did I pull on the string?



$m = 4\,kg$ $R = 2m$

$r = .8m$

$\alpha = 10\,rad/s^2$



Side View

A) Kinetics lens because we are looking at the disc's motion as an explicit function of time

$\omega_0 = 0$ $\qquad \alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \cancel{\omega_0}^{\,0}}{\Delta t}$ $\qquad 10\,rad/s^2 = \frac{\omega_f}{4s}$ $\qquad \boxed{\omega_f = 40\,rads/s}$

B) " "

$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ $\qquad \omega_{avg} = \frac{\omega_f - \cancel{\omega_0}^{\,0}}{2} = \frac{40\,rad/s}{2} = 20\,rad/s = \omega_{avg}$

$20\,rad/s = \frac{\Delta\theta}{4s}$ $\qquad \boxed{\Delta\theta = 80\,rad}$

A+

C) Rotational Dynamics ⟶ Dynamics: Angular acceleration of the wheel is caused by a torque. The torque is provided by a force acting on the radius of the pully.

$\tau = I\alpha$ $\qquad I = \frac{1}{2}mR^2$ $\qquad I = 4kg\,(2m)^2$ $\qquad I = 16\,kgm^2$

You pulled w/ 200N worth of force

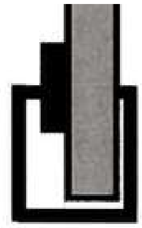
$\tau = 16kgm^2\,(10\,rad/s^2)$

$\tau = 160Nm$ $\qquad \tau = F_\perp r$ $\qquad F_\perp = \tau / r$ $\qquad F_\perp = \frac{160\,N\cancel{m}}{.8\cancel{m}} = \cancel{2}00N$

100 N

c) How hard did I pull on the string?

I am using a rotational kinematics lens to solve this because we are looking at rotational motion as an explicit function of time



Side View

☺

a.) Find $\vec{\omega}$

$\vec{\alpha} = \frac{\Delta \omega}{\Delta t}$ $\vec{\alpha} = 10/s^2 = \frac{\Delta \omega}{4s}$

rotational velocity
$\vec{\omega} = 40/s$ after 4s

$\frac{10}{s^2} \cdot 4s = \Delta \omega$

using rotational kinematics lens b/c looking at rotational motion over time

b.) $\vec{\omega} = \frac{\Delta \phi}{\Delta t}$ $40/s = \frac{\Delta \phi}{4s}$ $\Delta \phi = 80$? this is the right answer but how did you get it?

(ave

wheel has rotated 160radians after 4s

$\frac{40}{s} \cdot 4s = \Delta \phi$

c.) using rotational dynamics lense because torques, or turning forces cause rotational acceleration

$\sum \vec{T} = I \vec{\alpha}$

A

$\sum \vec{T} = 16 \text{ kgm}^2 \cdot 10/s^2$

$I_{disc} = \frac{1}{2} \text{mass} \times (\text{radius})^2$ lovely

$\sum \vec{T} = 160 \text{ Nm}$ → torque

$I = 4\text{kg} \times (2m)^2$

$\sum \vec{T} = F \perp r$ → Torque = force x perp radius.

$I = 4\text{kg} \cdot 4m^2$

$\sum \vec{T} = F(.8m) = 160 \text{ Nm}$

$I = 16 \text{ kgm}^2$

$\boxed{F = 200N}$ → 100N

2 (for an extra grade boost). I have two objects: a cart with low friction wheels, and a large circular hoop that rolls without slipping. I get them going on flat ground and then let them go at the base of a long hill. Which one gets further up the hill, or do they get to the same height? Fully support your answer.

Energy Lens

*same velocity

Car: Kinetic energy ⟶ Potential Energy

Hoop: Kinetic energy + RKE ⟶ PE

* The hoop will go higher up the ramp because both its linear & rotational energy will be converted to Potential energy

Using the energy lens (above) is the clearest way (in my opinion) to explain this answer. However, there are other ways, and several students used a rotational dynamics lens reasoning this way: both objects are moving forward and come to a stop because the force of gravity pulls them down. HOWEVER, the rotating hoop ALSO is spinning and thus requires a torque to rotationally accelerate it. This torque requires friction at the surface of the road. In order to decrease omega, this friction must be upwards, providing an upward force on the wheel… thus the hoop in slowing down, grips the road and "pushes itself" higher than the cart that is not rotating… or only has tiny wheels with negligible moments of inertia.

What’s your name?____