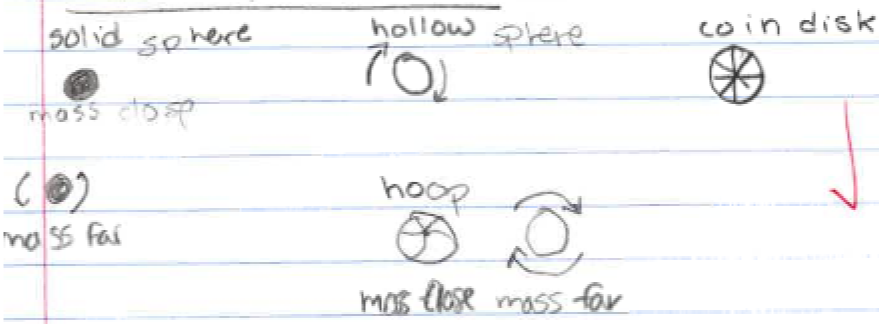
Problem Set #5 due beginning of class, Tuesday, Feb. 18

1) Section 4.5 Exercise 1, Ranking Several Objects *For these, we are looking at the radial distribution of the mass. The objects with the most mass further from the axis of rotation has the larger moment of inertia*

4.5 Exercise 1



solid sphere < hollow sphere < coin disk < standing hoop < disk < ...

2) Section 4.5 Exercise 2, Rolling Objects up a hill *Also see Assessment #6 solutions*

I_{hoop} : $\frac{2}{5}mr^2$
I_{disk} : $\frac{1}{2}mr^2$
$I_{solid\ sphere}$: mr^2

4.5 Exercise 2

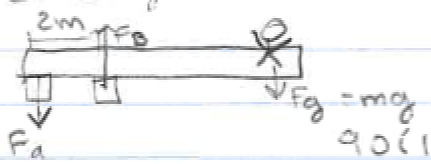
Rotational dynamics lense. Rotational acceleration is being caused by torques. The more inertia on a body, the harder it is to apply angular acceleration. The hoop will go further up the hill because it has the highest intertia so its harder for the Fg and friction to apply angular acceleration which would be how it slows down.

3) Section 4.6 Exercise 2, person on diving board

4.6 Exercise 2

Rotational Dynamics → $\sum\vec{\tau}=0$ and dynamics → $\sum\vec{F}=0$

The sum of $\vec{F}=0$ and there are forces causing an acceleration



$90(10) = 900N$

$T_A + T_g + F_B = 0$

$rF_A + rF_g + rF_B = 0$

$(2)(F_A) + (4)(-900) + 0(F_B) = 0$

$2mF_A (-3600\,Nm) = 0$

$\frac{2m\,F_A = 3600}{2}$

$F_A = \boxed{-1800N}$

$0(F_A) + 2(F_B) = 6(900N) = 0$

$2m(F_B) - 5400 = 0$

$F_B = \frac{5400Nm}{2}$ $\boxed{2700\,Nm}$

$\sum\vec{F} = F_A + F_B + F_G = 0$

$= -1800 + 2700 / -900Nm$

$= 0$

$\sum\vec{T} = T_A + T_B + T_G = 0$

$F_A(2) + F_B(0) + F_G(4) = 0$

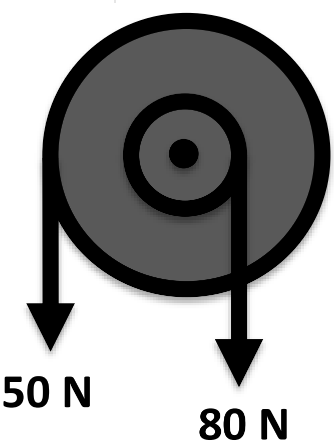
$-3600Nm + 3600Nm = 0$

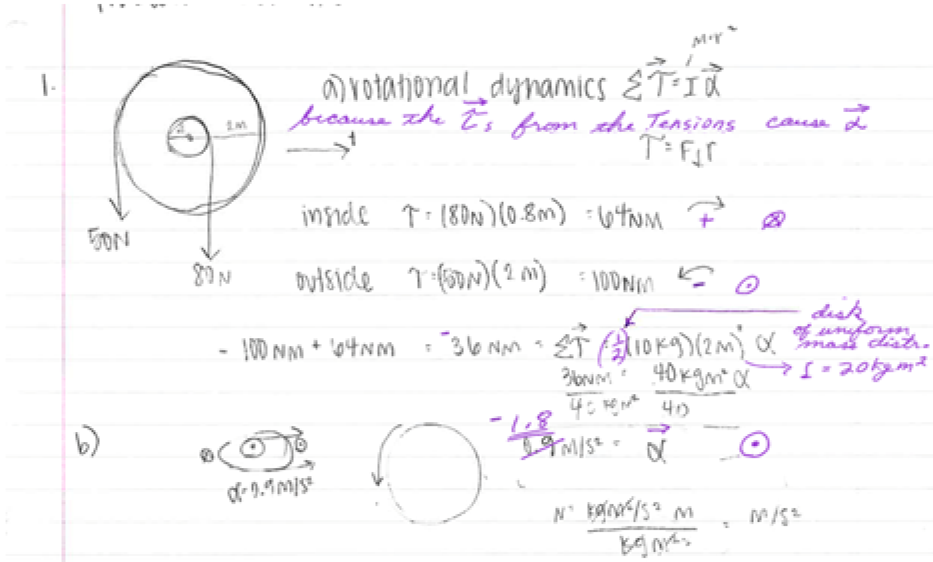
$0 = 0$

4) A 10 kg disc of radius 2 m has an attached pulley wheel of radius 80 cm. The wheel/pulley assembly is of uniform mass density and is free to spin on a low friction bearing. Strings are wrapped around the outside of the wheel and inner pulley as shown. With the wheel initially at rest, I pull on the string with the tensions indicated.

a) Please describe the subsequent motion of the wheel.

b) Quantify the resulting motion, and include the direction using the correct right-hand rule.




1\. a) rotational dynamics $\sum \vec{\tau} = I\vec{\alpha}$

because the $\vec{\tau}$s from the Tensions cause $\vec{\alpha}$

$\tau = F_{\perp} r$

inside $\tau = (80N)(0.8m) = 64Nm$ + ⊗

outside $\tau = (50N)(2m) = 100Nm$ − ⊙

$-100Nm + 64Nm = -36Nm = \sum \vec{\tau} = (\frac{1}{2})(10kg)(2m)^2 \alpha$ disk of uniform mass distr. $\rightarrow I = 20kgm^2$

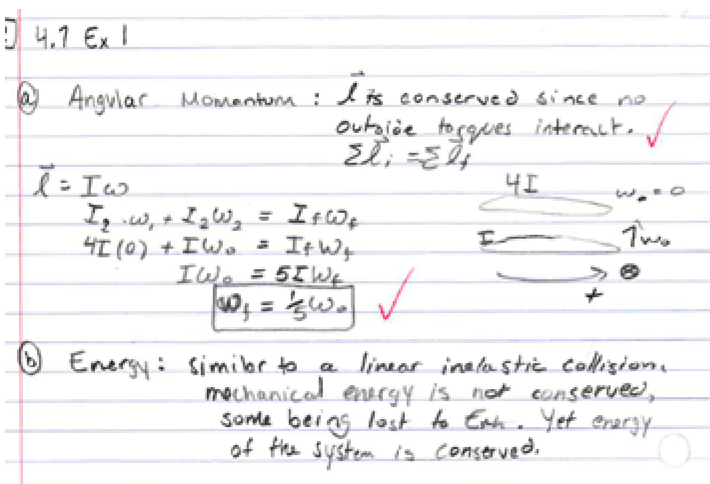
$\frac{36Nm}{40kgm^2} = \frac{40kgm^2 \alpha}{40}$

b) $\alpha = 0.9 m/s^2$

$\cancel{0.9}\ -1.8\ m/s^2 = \vec{\alpha}$ ⊙

$\frac{N \cdot m}{kg\,m^2} = \frac{kg\,m/s^2 \cdot m}{kg\,m^2} = m/s^2$

5) Section 4.7, Exercise 1. Dropping larger disk on rotating disk


4.7 Ex 1

a) Angular Momentum: $\vec{l}$ is conserved since no outside torques interact. ✓

$\sum l_i = \sum l_f$

$\vec{l} = I\omega$

$I_1\omega_1 + I_2\omega_2 = I_f\omega_f$

$4I(0) + I\omega_o = I_f\omega_f$

$I\omega_o = 5I\omega_f$

$\boxed{\omega_f = \frac{1}{5}\omega_o}$ ✓

4I $\omega_o = 0$

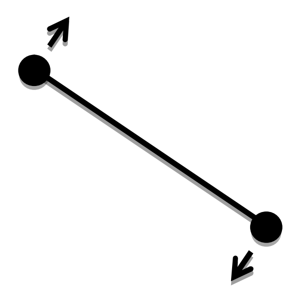
I ω_o

⊗ +

b) Energy: similar to a linear inelastic collision, mechanical energy is not conserved, some being lost to Eth. Yet energy of the system is conserved.

6) This is a variation of Section 4.7, Exercise 2

Two identical bodies are tied together with a string, are spinning in space about the center at angular speed, ω_i, when a motor at the center pulls them both inward such that the final diameter of their paths is 1/3 the original diameter, or, $d \Rightarrow \underline{1/3}\ d_i$. If this is in outer space, we can be sure there are no outside forces. We might consider conserving energy and/or angular momentum.



a) Is it possible for the angular momentum to change? If so how?

b) Is it possible for energy to change? if so, where did the energy go or come from?

Is it possible to conserve both angular momentum ***and*** energy? Let's find out!

c) What happens to the moment of inertia with this change? $I \Rightarrow$ ___ I_i

d) If we conserve angular momentum what should be the new angular velocity? $\omega \Rightarrow$ ___ ω_i,

e) Would this change the kinetic energy? If so, by what factor: $KE \Rightarrow$___ KE_i

f) Can we conserve angular momentum ***and*** energy? If not, which one must have changed, and where did that extra energy (or angular momentum) come from?

(14) 4.7 Ex) 2

a) - Momentum is conserved b/c there are no outside forces.
- Energy is always conserved, but KE would also be conserved here b/c there is no loss to heat ... maybe!
- Kinematics, $\vec{v}$ stays the same, so $\vec{\omega}$ would increase since the radius decreases
- There are no outside force. The only force is that of the motor pulling the mass's inward.

b) $I = mr^2$; $r = \frac{1}{3} r_0$, so $\boxed{I_F = \frac{1}{9} I_0}$

c) $L_0 = L_F$; Momentum lens
$I_0 \vec{\omega}_0 = I_F \vec{\omega}_F$; $I_F = \frac{1}{9} I_0$
$I_0 \vec{\omega}_0 = \frac{1}{9} I_0 \vec{\omega}_F$
$\vec{\omega}_0 = \frac{1}{9} \vec{\omega}_F$, $\boxed{\vec{\omega}_F = 9\vec{\omega}_0}$

d) Energy lens, there is KE_{rot}
$KE_0 = \frac{1}{2} I_0 \vec{\omega}_0^2$ $\frac{1}{2} I_0 \omega_0^2$; $\frac{1}{2}(\frac{1}{9} I_0)(9\vec{\omega}_0)^2$
$KE_F = \frac{1}{2} I_F \vec{\omega}_F^2$ $\frac{1}{2} I_0 \vec{\omega}_0^2$; $\frac{1}{2}(9) I_0 \omega_0 2$
$\boxed{KE_F = 9KE_0}$ I guess KE is not conserved   yes!

e) Energy lens
$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{2}(\frac{1}{9} I_0)(\vec{\omega}_F)^2$
$\frac{1}{2} I_0 \vec{\omega}_0^2 = \frac{1}{18}(I_0)(\vec{\omega}_F)^2$ so $\boxed{\vec{\omega}_F = \sqrt{2}\,\vec{\omega}_0}$

f) Yes, momentum lens
$\boxed{L_F = \frac{9}{9} L_0}$

g) I think that L is conserved and KE_{rot} is not, because work must be done in order to move the masses inward. yes!

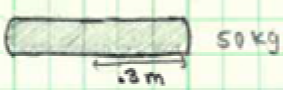
7) You have an ax to grind, and you decide to grind it on the outer rim of a round 5 kg stone grinding wheel of uniform thickness and radius 30 cm. The coefficient of friction between steel and stone is 0.3. You spin the wheel up to 1000 rpm with a 100 W motor.

a) What is the angular velocity of 1000 rpm?

b) How long does it take to spin the wheel up to 1000 rpm? What lens do you use?

c) Then I push the ax against the wheel with a force of 100 N and the sparks fly! But as soon as you start, the electricity goes out and the wheel is spinning freely without power. What is the angular acceleration of the wheel as you push against it with the ax?

4.



$\mu = 0.3$ spun @ 1000 rpm w/ 100 W motor

Lens: Rotational Dynamics because a force is causing acceleration

$I = \frac{1}{2}(50\text{kg})(.3\text{m})^2 = 25 \times .09 = 2.25$

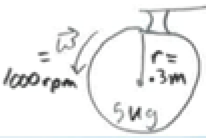
1 rpm = $\frac{2\pi}{60}$ rad/s

$\omega = \frac{d\theta}{dt}$

a) $1000\text{ rpm} \cdot \frac{2\pi/60\text{s}}{1\text{ rpm}} = 200\cancel{0}\pi/6\cancel{0}\text{s}$ ✓
$\boxed{33.33\ \pi/\text{s}}$

Because pi is about 3, ω ~ 100/s or 100 radians/s.

Noting that I = ½ mr^2 for a solid disk of uniform thickness,



$\mu = .3$

a) Energy lens, b/c the work done by the motor is converted to the rotational kinetic energy of the wheel

$KE_{rot} = \frac{1}{2} I\omega^2$; $I = \frac{1}{2}mr^2$ (disk)

$\vec\omega = \frac{1000 \text{ rot}}{\text{min}} = \frac{2000\pi}{60s} \approx 100/s$

$KE_{rot} = \frac{1}{2}\left(\frac{1}{2} 5kg (.3m)^2\right)(100/s)^2$

$KE_{rot} = 1{,}125 J$

$P_{motor} = 100W = \frac{100J}{s}$; $1125J \cdot \frac{s}{100J} \approx \boxed{11.25 s}$

b) Dynamics lens, b/c there are forces and accelerations.

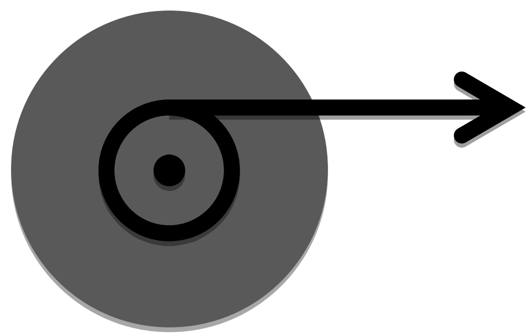
$\sum\vec\tau = I\vec\alpha$ (For Fixed I)

$\tau = F \cdot r = F_F \cdot r = \mu F_N \cdot r$

$\tau = (.3)(100N) \cdot (.3m) = 9Nm$

$\vec\alpha = \frac{9Nm}{\frac{1}{2}(5kg)(.3m^2)} \approx \boxed{40/s^2}$

8) A concrete flywheel of uniform thickness has a mass of 50 kg and a radius of 40 cm. If I pull on the string with a force of 100 N that is wound around a pulley of radius 16 cm.

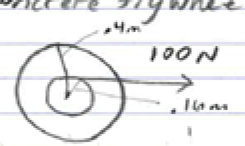


a) what will be the angular velocity, ω after 10 s? Remember, no credit without lens.

b) What if instead of pulling for 10 s, I ask you to find the final angular velocity, ω after pulling 2 meters of string. Would this change the lens that you would look through? Please find the final angular velocity, ω.

c) Just as we did in linear problems, for problem (b) above, knowing the final ω, please find the average angular velocity, the time it took to pull the string and the average power that the wheel/ax produced in thermal energy as you slowed the wheel.

Concrete flywheel



m=50 kg

Rotational dynamics: Forces and torque acting to cause α.

$\sum\vec F = m\vec a$ $\vec\tau = I\vec\alpha$

$I = \frac{1}{2}mr^2$

Kinematics: function of time

(a) $F(r) = [mr^2\frac{1}{2}]\alpha$

$100N(.16m) = [(50kg)(.4m)^2\frac{1}{2}]\alpha$

$16N\cdot m = \frac{1}{2} 50kg \cdot .16m\ \alpha$

$\alpha = 4\ 1/s^2$

$\omega = \int \alpha\, dt$

$= 4 s^{-2}(10s)$

$\boxed{\omega = 40 /s}$

9) Chapter 5.0, Exercise 1, You see something moving in a circle.

③ • Yes, from dynamics lens, $\vec{a}_c \neq 0$, so there must be a force acting on the rock.

• Dynamics lens → there are forces + accelerations

$\Sigma\vec{F} = m\vec{a}_c$; $\vec{a}_c = \frac{v^2}{r} = \frac{(10\,m/s)^2}{20m} = \boxed{5\,m/s^2}$

• $\Sigma\vec{F} = m\vec{a}_c$, $F = (10kg)(5\,m/s^2) = \boxed{50N}$

• I have no idea b/c I can't see what is going on.

• Tension Force

$F_T = m\vec{a}_c = \boxed{50N}$

• The velocity of the rock is always tangential to its circular path. If the string breaks, the rock no longer has a centripetal acceleration; no force keeping its motion circular. So, the rock will continue in a straight line.

• Must be friction.

$\Sigma\vec{F} = m\vec{a}_c$; $F_f = m\vec{a}_c$; $\mu F_N = m\vec{a}_c$

$\mu = \frac{m\vec{a}_c}{F_N} = \frac{m\vec{a}_c}{mg} = \frac{\vec{a}_c}{g} = \frac{5\,m/s^2}{10\,m/s^2} = \boxed{\frac{1}{2}}$

- This is static friction b/c the tires are no sliding, they are rolling.

10) Chapter 5.1 Exercise 1, Lunar period!

④ - Dynamics lens, involves forces and acceleration

$\frac{385{,}000\,km}{6400\,km} = 60$. Radius from Earth to moon is 60 times greater than Earth's center to surface. Inverse square law says that gravity will be equal to $10\,m/s^2 \cdot \frac{1}{r^2}$

So $10\,m/s^2 \cdot \frac{1}{60^2} = \boxed{.002\,m/s^2}$. - This acceleration is caused by gravity

- Moon's acceleration is the same as that. Moon just has tangential velocity, which is why it doesn't crash into Earth. There is just one force acting on the moon. This doesn't change.

- a_{grav} is directed toward center, so it is equivalent to centripetal acceleration

$a_{grav} = a_c$

$.002\,m/s^2 = \frac{v^2}{385{,}000{,}000\,m}$ → $\boxed{V = 880\,m/s}$

Circumference is distance

- Period = time of orbit. Kinematics lens, involves motion and time.

$\Delta x = vt$ → $\frac{\Delta x}{v} = t$ → $\frac{2\pi(385{,}000{,}000\,m)}{880\,m/s} = \boxed{2748935} = \boxed{31\,days}$

11) Chapter 5.2, Exercise 2, Lower Earth Orbit

⑤ Dynamics lens - There are forces + accelerations — I made

At sea level, $R_S = 6{,}400\,km$

At LEO, $R_L = 6{,}560\,km$

So, $R_L = (1.025)R_S$

So $F_L = (1.025^2)F_S = (1.051)F_S$

So $\boxed{\vec{a}_L = 1.051\,\vec{a}_S}$, so a factor of 1.051 is roughly the same

- Kinematics lens b/c motion of sat. is an explicit function of time.

$\vec{a}_c = \frac{v^2}{r}$, $10\,m/s^2 = \frac{v^2}{6560\,km}$,

$v = \sqrt{(6560000\,m)(10\,m/s^2)}$, $\boxed{v = 8100\,m/s}$

$T = \Delta t = \frac{\Delta d}{v} = \frac{2\pi r}{8100\,m/s} = 1.5$ hours

- 24 hours in a day, going around Earth 1.5 hours at a time so 16 times