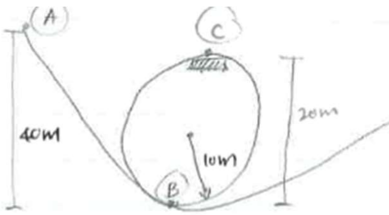
Problem Set #7 due beginning of class, Monday, Feb. 24.

1) Chapter 5.3, Do exercise 1 and 2, but don't hand it in. Does it stay in the bucket? Answers at end of chapter.

2) Chapter 5.3, Exercise 4, Loop the loop

7) (5.3, #4) You go on a R=10m loop ride, cart is let go on low friction track, pulled downhill by gravity. I have mass: 70 kg.



a) Start from vertical height h=40m. I use an energy lens since I see E_p turning into E_k as the cart moves down the loop. $\sum E = E_p + E_k$, $E_p = mg\Delta h$, $E_k = \frac{1}{2}mv^2$

$E_{p(A)} = E_{k(C)} + E_{p(C)}$	$E_{p(A)} = E_{k(B)}$
$mgh_A = \frac{1}{2}mv_c^2 + mgh_c$	$mgh_A = \frac{1}{2}mv_B^2$
$V_c = \sqrt{2g(h_A - h_c)}$	$V_B = \sqrt{2gh_A}$
$V_c = \sqrt{2(10\,m/s^2)(40m - 20m)}$	$V_B = \sqrt{2(10\,m/s^2)(40\,m)}$
$V_c = 20\,m/s$	$V_B = 28.28\,m/s$

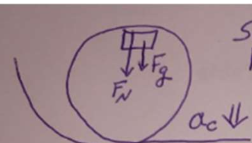
Then I use a dynamics lens since I see something moving in a circle, I know $a_c = \frac{v^2}{r}$, so: →

b) and c) : as you reduce your speed, the acceleration is reduced so the force producing this acceleration must also be reduced. However, the total force at the top of the loop can not get smaller than the force of gravity, which is when the normal force is zero! Thus you can't start at the same height as the top, because if you did, you'd have no speed (kinetic energy) at the top of the loop and you would accelerate off the track and fall to your death!. The minimum speed possible will result in a $a_c = g$. I did this in class on the board:

$$a_c = \frac{v^2}{r} = g;\ v^2 = rg$$

How high do we need to start to achieve this speed at the top? We will use our energy lens because $E_g \Rightarrow E_k$, or $mg\Delta h = \frac{1}{2}mv^2$. You should find that mass and gravity cancels leaving the extra height: $\Delta h = \frac{1}{2}r$, which is $5m$ meaning you have to start from an initial height of 25 m.

Scale Reads F_N



$a_c \Downarrow$ $+\Downarrow$

$\Sigma\vec{F}$ F_g F_N

$V = 20\,m/s$

$a_c = \frac{v^2}{r} = 40\,m/s^2$

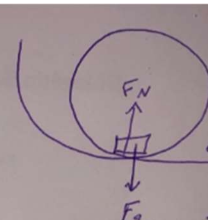
$a_c = \frac{\Sigma\vec{F}}{m}$

$+ma_c = +F_g + F_N$

$F_N = ma_c - F_g$

$= 2800N - 700N$

$F_N = 2100N$ at top



$a_c \Uparrow$ $+\Uparrow$

F_g F_N $\Sigma\vec{F}$

$V = 28\,m/s$

$a_c = \frac{v^2}{r} = 80\,m/s^2$

$a_c = \frac{\Sigma\vec{F}}{m}$

$+ma_c = F_g + F_N$

$F_N = ma_c + F_g$

$= 5600N + 700N$

$F_N = 6300N$ @ Bottom

@ the bottom, you feel "heavier" because there is a greater F_N

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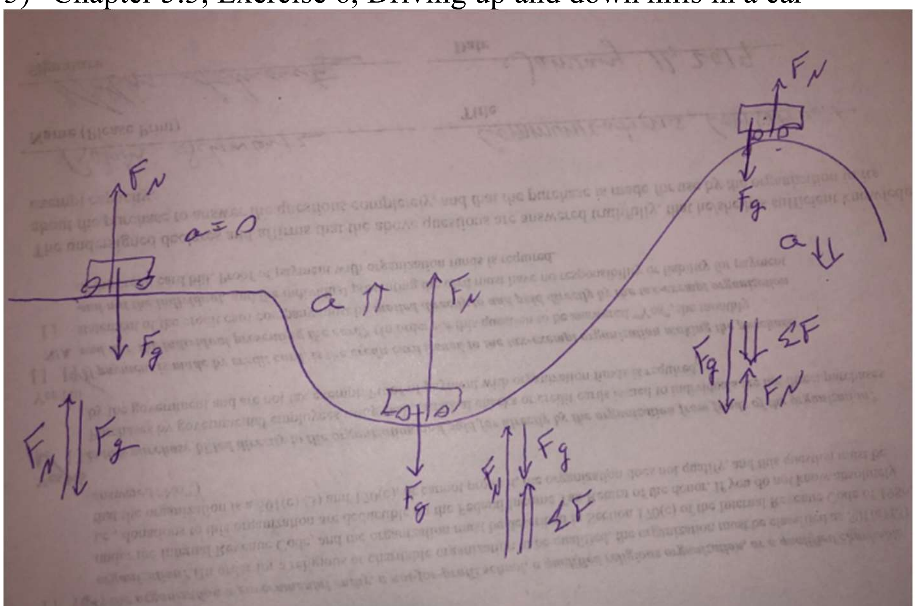
$$a_c = \frac{v^2}{r} = g;\ v^2 = rg$$

How high do we need to start to achieve this speed at the top? We will use our energy lens because $E_g \Rightarrow E_k$, or $mg\Delta h = \frac{1}{2}mv^2$. You should find that mass and gravity cancels leaving the extra height: $\Delta h = \frac{1}{2}r$, which is $5m$ meaning you have to start from an initial height of 25 m.

3) Chapter 5.3, Exercise 6, Driving up and down hills in a car



F_g

b) From a dynamics lens ($\vec{F}$'s and $\vec{a}$'s), we can see that $\Sigma\vec{F} = 0$ in case 1, so you feel completely normal. In case 2, $\vec{a}$ is upward, so you feel "heavier". In case 3, $\vec{a}$ = downward, so you feel "lighter". Does this match experience?

Again, you experience the greater normal force as "feeling heavier".

4) Chapter 5.3, Exercise 7, Do you weigh the same at the equator?

Clearly a dynamics problem because the forces (normal force and gravity) cause acceleration… But the acceleration on the pole is zero (and so you are in equilibrium, and $F_N = F_g$) because I have no velocity there. On the equator, the centripetal acceleration is inward because of the uniform circular motion. You will need a good FBD here to show that at the equator, $F_N < F_g$, where at the pole the forces are equal and opposite. Also, do a good vector addition diagram to show that if the acceleration is inward, the net force is inward, and thus $F_N < F_g$. Correct answer: b)

5) Chapter 6.0, carefully consider Examples 1, 2, and 3. Then do the following:
What if there is a coefficient of friction $(\mu_d = 0.1)$ on the 1 kg mass as is slides across the horizontal surface?

a) How would this change the energy considerations in Example 1? Find the new speed of the system as it hits the ground 1 m below. Then find the time to fall and the acceleration.
We covered this during class on the 19th. Example 1 uses an energy lens to look at the system. The potential energy of the hanging mass is converted to the kinetic energy of the two-mass system. When you add friction, some of the potential energy is "lost" to thermal energy = $W_{friction} = F_f * \Delta x$

b) How would this change the dynamics considerations of the system (Exercise 2)? Find the new acceleration directly (is it the same as you found above?) and tension in the string. It will be helpful to first watch the systems/dynamics video for Monday's class.
This is the system dynamics lens. Here we simply include the force of friction as an additional force.

c) How would this consideration change the method of simultaneous equations (exercise 3)? This is using the dynamics lens on each mass. We need to include the force of friction on the mass that is sliding. Be careful of sign!

6) Chapter 6.1, Example 3
As we demonstrated in class, we can use a dynamics lens to see that the tension in the string is less than the force of gravity because the acceleration is downward. We can use kinematics lens to show that Wheel A has a higher linear velocity of the hanging mass for the same rotational velocity of the wheel, and then an energy lens because the gravitational potential energy of the system turns to rotational and linear kinetic energy. Using this energy lens, we can show that Wheel A has a larger kinetic energy when it hits the ground, and thus has a higher linear acceleration, and thus has a lower tension.

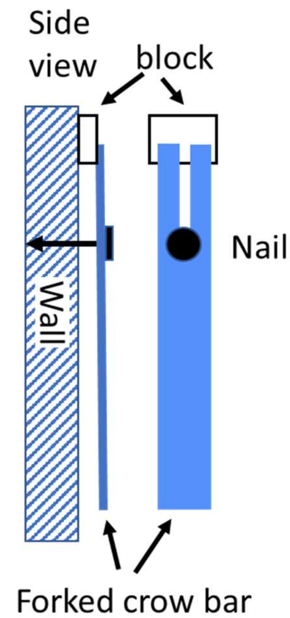
7) Chapter 6.1, Example 4
- As we demonstrated in class, this is a very similar problem to Example 3 above. Using and energy or dynamics lens, we should show that mass cancels in all terms, so mass doesn't matter… and indeed mass has never made a difference in a system falling under the force of gravity.
- You should also be able to show that radius will cancel on both sides of the equation if you use an energy or dynamics lens. For instance, use and energy lens because gravitational potential energy is converted to linear and rotational kinetic energy. Then notice that omega = v/R, and that this R is the same as the R in the moment of inertia. You should see that the R cancels. What this means is that if you double the radius of the wheel, it will have the same final velocity when it hits the ground but be rotating at half the rotational velocity. However, it's rotational energy will still be the same because the moment of inertia has increased.
- If you drill out the center and fill it with something more dense (at a low radius), now you have lowered the portion of energy turned to rotational kinetic energy. Thus, there will be a greater portion of potential energy turned to linear kinetic energy. The final velocity will be greater and the downward acceleration will be greater.

8) I use the device at right to pry a nail out of a wall. At right is what you see looking directly at the wall. At left is a side view, looking along the wall. The distance between the nail and the block is 5 cm. I pull with a force of 200 N on the bottom end of the crow bar, which is 15 cm below the nail.

a) How much force did this put on the nail?

b) What force is put on the block?

Please see Assessment #7 solutions



9) Two identical planets orbit the same star in a circular path. But planet B is twice as far as planet A. That is:

$R_B = \underline{2}R_A$

Please find the following ratios:

a) Which planet has greater gravitational attraction to the sun, or are they the same?
or, $F_B = \_\_F_A$

b) Which planet has a greater acceleration or are they the same?
or, $a_B = \_\_a_A$

c) Which planet has a greater speed, or are they the same?
or, $v_B = \_\_v_A$

Please see Assessment #7 solutions