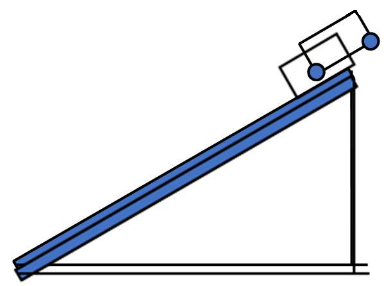
PS#9 Due in Class Monday, March 9 Please pay good attention to describe the lens you are using and explain your method.

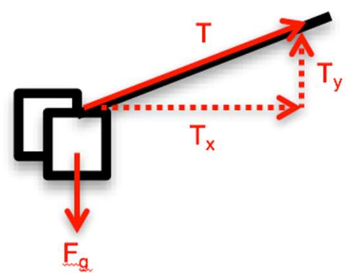
**** Make sure to consider the direction of acceleration to inform your choice of axis. Do you remember how to pick a good axis?

1. Please read section 7.2 and consider the cart of mass m_o at right, released from rest on a low friction surface.



   - Please find the resultant force on the cart in terms of constants that we know. Clearly outline your approach. We recognize this as a dynamics problem because the forces cause acceleration down the ramp. We do a good free body diagram and note that the interesting directions are parallel and perpendicular to the inclined surface because the cart accelerates parallel to this surface. We decompose the force of gravity into a parallel component and a perpendicular component. We see that the parallel component is about half of the full force of gravity or about ½mg.
   - Please estimate the acceleration down the track. Recognizing that F=ma, the acceleration down the ramp is about half of gravity, or about 5 m/s^2.
   - Repeat the above two questions if there is a coefficient of dynamic friction of 0.3 between the cart and the track. We need to find the normal force. If we did a good job with the FBD, we should see it's about 0.85 mg, yielding a force of friction of about 0.25 mg. Assuming the cart is moving downward, this force of friction is up the ramp, leaving only about 0.25mg down the ramp, for an acceleration of 2.5 m/s^2.
   - What coefficient of friction would be necessary for the cart to move at a constant speed? Constant velocity means it's in equilibrium and the sum of the forces = 0. So, the force of friction needs to be about ½mg. Given a normal force of about 0.85, the frictional coefficient would need to be about 0.6.
   - If the block had wheels with considerable mass, how would this affect the acceleration? Why? I'd prefer to use an energy lens because when we add mass to the wheels, we'd see that in the energy transformation from potential energy to kinetic energy, some linear kinetic energy would be sacrificed to provide rotational kinetic energy of the wheels. Using a dynamics/rotational dynamics lens, we realize that in order to rotationally accelerate the wheels, torque is necessary requiring tangential force on the wheels in the "uphill" direction… this force decreases the acceleration of the cart.

2. You are watching the fuzzy dice from the rearview mirror. As you take off on level ground, it makes an angle as shown at right.



   a) state how you will inform your choice of axis.

This is a dynamics lens because we have forces acting on the dice causing them to accelerate. We write $\sum \vec{F} = m\vec{a}$ and identify forces and direction of acceleration. Because the acceleration is horizontal, we break things up into horizontal and vertical components.

   b) Estimate the acceleration of the car.

We see that $T_y = F_g$ because $a_Y = 0$. This we can see that the x component of tension (which is what's accelerating the dice in the x direction) is about twice $3*T_y = 3*F_g$ So the acceleration must be about 3 gravities or 20 m/s^2.

c) What must be the coefficient of friction of your tires for this to happen?

Because the normal force on the car = F_g between the car and road (because there's no vertical acceleration), we can see that the coefficient of friction would have to be an extraordinarily high value of 3.0 in order to attain such a high acceleration.

d) Is this realistic?

It's possible, but very unlikely, and not possible for regular tires and cars.

e) If the mass of the dice is 100 g, what is the tension in the string?

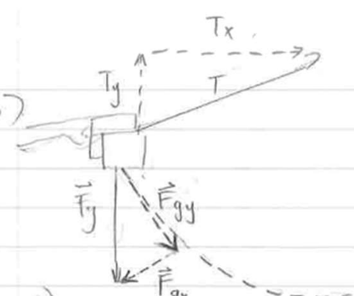
Looking back at the work we did for b) we can see that the tension should be a little more than $3*F_g$ or about 35 N.

3. Consider the fuzzy dice above. Now the car is stationary and you are sitting it in. You grab the dice and pull them to one side exactly as in the diagram above. Then you let go of them.
   a) **** Choose a good axis. Is the direction of acceleration the same as above? State how this direction will inform your choice of axis.

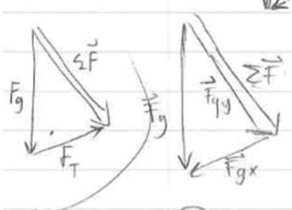
Now the acceleration is tangential, perpendicular to the radius, so our axes are radial and tangential, so we keep tension as a single force and decompose gravity into parallel (tangential) and perpendicular (radial) components.

   b) Again find the acceleration of the dice with direction.
   c) Again, if the mass of the dice is 100 g, please find the tension in the string. Is it the same as the string above? Why might this make sense?

B 3)



a) I use a dynamics lens since I see forces causing acceleration and since I see a body moving in a circular path; it has a_{cp} caused by some force; The $\vec{a}$ this time is in the direction of $\Sigma\vec{F}$ along the circle



b) I use the same lens for the same reason.

$\vec{a}_{radial} = 0$

$\therefore F_{gx} = T$

$\therefore \Sigma F = F_{gy}$

$F_{gy} \sim 0.9 F_g = 0.9(mg)$

$\Sigma \vec{F} = m\vec{a} = F_{gy}$

$0.9\cancel{m}g = \cancel{m}\vec{a}_{tang}$

$\vec{a} = 0.9g$

$\boxed{\vec{a} = 9\ m/s^2}$

good

c) I use the same lens for the same reason.

I don't understand

$T = F_{gx} \sim 0.4F_g$

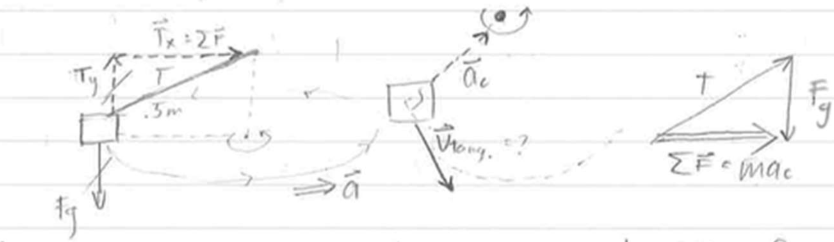
$T = 0.4(mg)$

$T = 0.4(0.1\ kg)(10\ m/s^2)$

$\boxed{T = 0.4\ N}$

4. Consider the fuzzy dice above. Now you are holding them from the end of the 50 cm string, and spinning the dice around in a circle. The path of the dice is a circle in the horizontal plane. Estimate the speed of the dice and the tension in the string.

(*) A 4)



I use a dynamics lens since I see forces causing acceleration. $\Sigma \vec F = m\vec a$.

$\vec a_y = 0 \Rightarrow F_g = T_y$

$\therefore T_x = \Sigma \vec F$ $\quad T_x \sim 3T_y$

$\therefore T_x \sim 3F_g$

$T_x = \Sigma F = m\vec a_c = m\frac{v^2}{r}$

$3F_g = m\frac{v^2}{r}$

$3mg = m\frac{v^2}{r}$

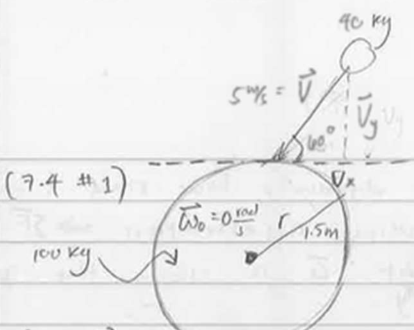
$3gr = \vec v^2$

$\vec v = \sqrt{3gr}$

$\vec v = \sqrt{3(10\ m/s^2)(0.5m)}$

$\boxed{\vec v = 3.87\ m/s}$

5. 7.4 Exercise 1, a child jumps onto a carousel.



$\vec V_x = (5\ m/s)\cos 60$ $\quad \vec V_y = (5\ m/s)\sin 60$

$\vec V_x = 2.5\ m/s$ $\quad \vec V_y = 4.33\ m/s$

A-7) (7.4 #1)

a) I use an angular momentum $\vec L$ lens since I see that w/ no external torques, then $\vec L$ is conserved.

$\Rightarrow \Sigma\Delta\vec\tau = 0 \Rightarrow \vec L_i = \vec L_f$ $\quad \vec L = I\vec\omega$

$\vec V = r\vec\omega$

$\vec\omega = \frac{\vec V}{r}$ $\quad \vec L = I\vec\omega$

$\vec L = mr^2\left(\frac{\vec V}{r}\right) = mr\vec v$

$I_{point\ mass} = mr^2$ $\quad I_{disk} = \frac12 mr^2$

$\vec L_{child} + \vec L_{disk} = \vec L_{child + disk}$

$m_c r\vec v_x + \left(\frac12 m_d r^2\right)\vec\omega_0 = \left(m_c r^2 + \frac12 m_d r^2\right)\vec\omega$

$\vec\omega = \frac{m_c r\vec v_x}{m_c r^2 + \frac12 m_d r^2} = \frac{(40kg)(1.5m)(2.5\ m/s)}{(40kg)(1.5m)^2 + \frac12(100kg)(1.5m)^2}$

$\vec\omega = \frac{150\ kg\ m^2/s}{202.5\ kg\ m^2} = \boxed{0.74\ rad/s}$

(*) b) I use an energy lens since I see transformations of energy as the child jumps onto the carousel. $\Sigma E_i = \Sigma E_f$.

$\Sigma E_i = KE_{child} = \frac12 mv^2 = \frac12(40kg)(5\ m/s^2) = 500\ J$

$\Sigma E_f = KE_{child+disk} = \frac12\left[(40kg)(1.5m)^2 + \frac12(100kg)(1.5m)^2\right](0.74\ rad/s) = 75\ J$

oops, while this student did a great job, there is a little mistake here. The final kinetic energy is $E_{K-Rot} = \frac{1}{2}I\omega^2, where\ I = \frac{1}{2}m_{disk}R^2 + m_{girl}R^2$. This student forgot to square the rotational velocity. I get a final rotational kinetic energy of only 55 J., We see that close to 90% of the kinetic energy is transformed to thermal energy. Also, please note that although the kinetic energy is correctly calculated, the square for the speed needs to be outside the parenthesis.

Angular momentum lens

c) The collision would decrease the rotation rate, since the carousel is rotating in the opposite direction that the child would hit and provide angular momentum.

$$\vec{L}_o = \vec{L}_f$$

$$mvr_\perp - I\vec{\omega} = L_f$$

d) momentum lens. Momentum is always conserved since there are no outside forces. The girl's momentum is transferred to the Earth, but since the Earth's mass is so large, velocity is negligable.

e) momentum lens because no outside forces.

$$p_o = p_f$$

$$\overset{0}{\cancel{p_{co}}} + p_{po} = p_f$$

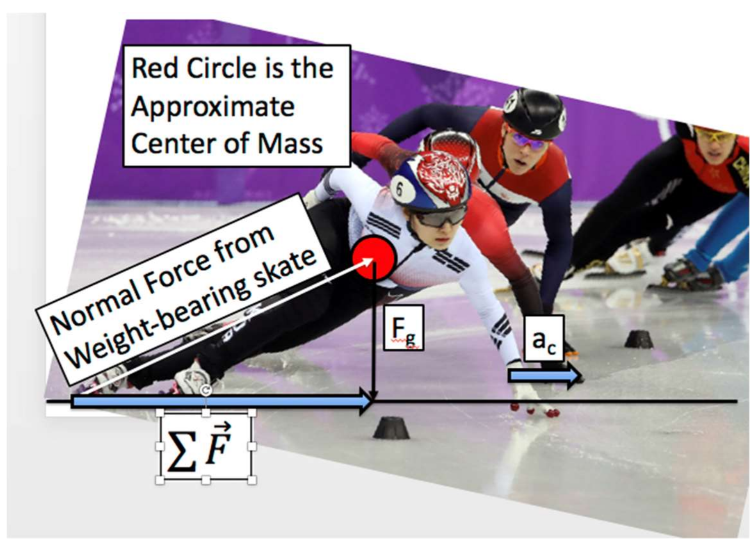
$$mv_{po} = m_{c+p} v_f$$

$$40\,\text{kg}(5\,\text{m/s}) = 140\,\text{kg}\, v_f$$

$$v_f = 1.4\,\text{m/s} \text{ southwest}$$

Also, again note that momentum is conserved (*all components* of momentum) independent of conserving the angular momentum (where only the tangential component contributes).

6. The 60 kg speed skater at right is executing a turn.
   a) If she is standing on one leg at this moment, estimate the force on her leg. Is this a lot of force? Could you stand on one leg with this much force on your leg?



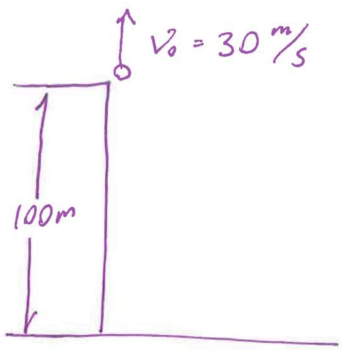
A dynamics lens is needed for this because the forces acting on this skater cause the (centripetal) acceleration. We write $\sum \vec{F} = m\vec{a}$, and make a free body diagram… clearly noting that she is not in equilibrium because she's moving in a circle, so we label the direction of the centripetal acceleration (into the center of the circle). We make sure then that we add the normal force from her leg and the force of gravity such that the $\sum \vec{F}$ is in the same direction as the acceleration. We see that the normal force is about 2.5 times as large as the force of gravity, so if her mass is 60 kg, the force of gravity on her is 600 N, and the normal force is about 1500 N. I think it would be very hard to hold 2.5 times your weight on one leg.

b) The radius of curvature of these tracks is 15 m. Estimate the speed of this skater. Looking at the geometry above, we see that $\sum \vec{F}$ is about twice as large in magnitude as the force of gravity, or about 1200 N, yielding an acceleration of 2*g = 20 m/s^2… wow. This is her centripetal acceleration = v^2/R. Solving for v, we get $\sqrt{aR}$ =(300 m^2/s^2)$^{.5}$ ~ 17 m/s… like 38 mph, that's pretty fast no?

7. 7.6 Exercises 1 and 2, deriving our two kinematic equations. These are covered in the videos, and you don't have to hand them in, but it's a good exercise to do them in order to know where the formulas come from.

8. 7.6 Exercise 3, Throwing a rock upwards off the edge of a cliff.

I use a kinematics lens because we have motion: an explicit f(t)



$$y(t) = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$0 = \underbrace{100m}_{c} + \underbrace{30\tfrac{m}{s}}_{b}\, t \underbrace{- 5\tfrac{m}{s^2}}_{a}\, t^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-30\tfrac{m}{s} \pm \sqrt{(30\tfrac{m}{s})^2 - 4(-5\tfrac{m}{s^2})(100m)}}{2(-5\tfrac{m}{s^2})}$$

multiply through by (−1)

$$= \frac{30\tfrac{m}{s} \mp \sqrt{900\tfrac{m^2}{s^2} + 2000\tfrac{m^2}{s^2}}}{10\tfrac{m}{s^2}}$$

$$= 3s \pm \sqrt{29}\ s$$

$$\approx 3s \pm 5.4s = -2.4s,\ 8.4s$$

The negative value.. given this trajectory, if we went backwards in time, it would be at the bottom of the cliff, moving upwards at about 54 m/s … = 30 m/s + g(2.4s)

But we didn't need the quadradic equation. We knew all along how to find time:

$\Delta X = V_{ave}\,\Delta t$ $\quad \Delta t = \dfrac{\Delta X}{V_{ave}}$ $\quad V_{ave} = \dfrac{(V_0 + V_f)}{2}$

we can use this given constant acceleration (g)

We can find V_f using an energy lens because

$E_p \Rightarrow E_K$ $\qquad E_o = E_f$

$$E_K + E_p = E_K$$

$$mgh_o + \frac{1}{2}mv_o^2 = \frac{1}{2}mv_f^2$$

$$V_f = (v_o^2 + 2gh_o)^{\frac{1}{2}}$$

$$= \left[(30\,m/s)^2 + 2(10\,m/s^2)100\,m\right]^{\frac{1}{2}}$$

$$\approx 54\,m/s,$$

$+\uparrow$

$$V_{ave} = \frac{(30\,m/s + -54\,m/s)}{2} \approx -12\,m/s \qquad \Delta x = -100\,m$$

$$\Delta t = \frac{\Delta x}{V_{ave}} = \frac{-100\,m}{-12\,m/s} \approx \underline{\underline{8.3\,s}}$$ ✓

9. 7.6 Exercise 4, Catching the Bus.

PS #6

#1 - Catching Bus

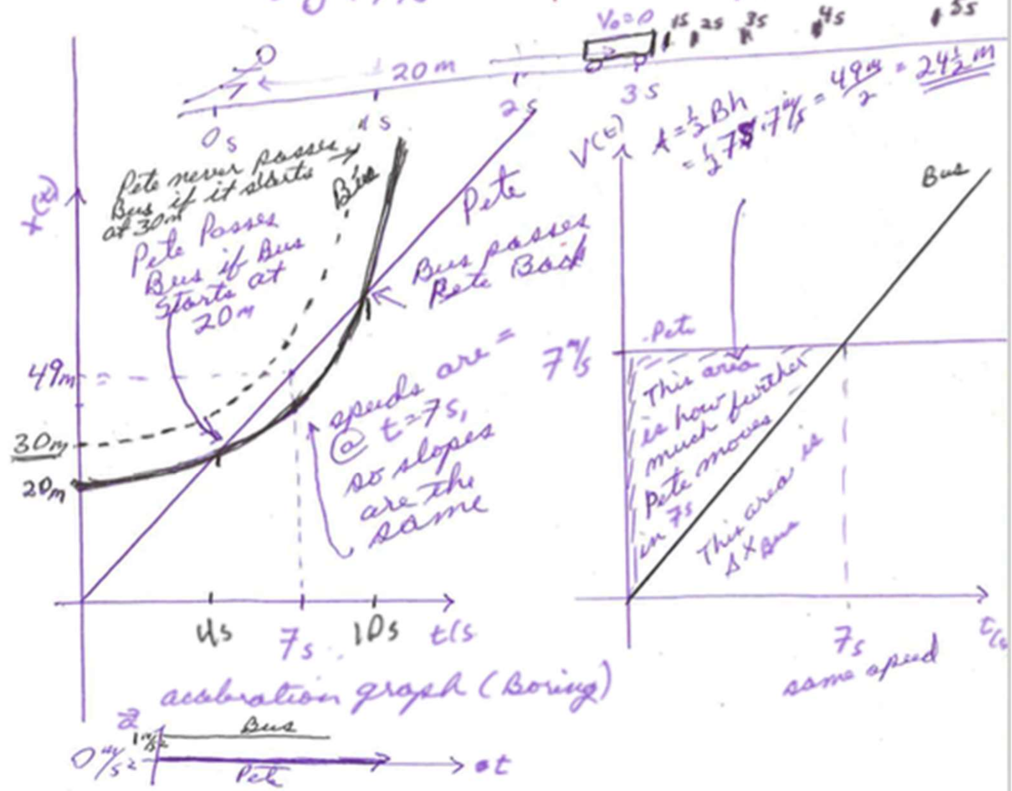
Pete: $V_P = 7\,m/s$ = const, $X_0 = 0$

Bus: $V_{0B} = 0$, $a_B = 1\,m/s^2$, $X_0 = 20m$, $V = a_B t$

Kinematics - because we are dealing with exclusive use of position and its time derivatives as an explicit function of time. In particular: $X_P(t) \stackrel{?}{=} X_B(t)$ when and if are our displacements the same

Pete: $X = X_0 + Vt$
$= 0 + 7\,m/s\;t$

Bus: $X = X_0 + V_0 t + \frac{1}{2}at^2$
$= 20m + 0 + \frac{1}{2}\,m/s^2\;t^2$



6 7.6 Ex. 4

Lens: kinematics, we have velocity and $\vec{a}$ as a fn of time.

~~$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$~~

Person

$x(t) = x_0 + V_0 t + \frac{1}{2} a t^2$

$x(t) = 0 + 7 m/s\, t + 0$

$x_p(t) = 7 m/s\, t$

Bus

$x_B(t) = 20m + 0 + \frac{1}{2}(1 m/s^2) t^2$

$x_B(t) = 20m + \frac{1}{2} m/s^2 t^2$

$7 m/s\, t = 20m + \frac{1}{2} m/s^2 t^2$

$0 = \frac{1}{2} m/s^2 t^2 - 7 m/s\, t + 20m$

$t = \frac{7 \pm \sqrt{49 - 4(\frac{1}{2})(20)}}{1} = 7 \pm \sqrt{9}$

$t = 7 \pm 3$

$t = 4, 10$

$x_p(t) = 7 m/s\, t$

$x_p(4) = 7 m/s (4s) = \boxed{28 m}$ @ 4s

$x_p(10) = 7 m/s (10s) = \boxed{70 m}$ @ 10s

You catch the bus

10. 7.6 Exercises 5 – 7 (Pulling sled, Hitting a baseball, Torque on a wheel.

7.6 Exercise 5. We would solve this problem exactly as we did before we used trigonometry. The only difference is now we could calculate the components rather than just eyeball (estimate) them. Of course, we recognize this as a dynamics problem whereby the acceleration is horizontal, so we choose x-y components and break the tension into horizontal and vertical components.

$T_x = T\cos(30^o) \sim 40\ N*(0.866) = 34.6\ N$

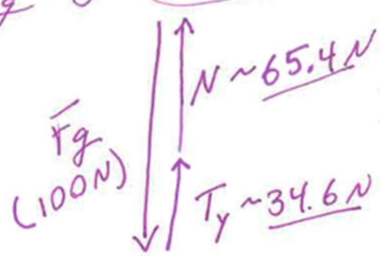
$T_x = T\sin(30^o) \sim 40\ N*(0.5) = 20\ N.$

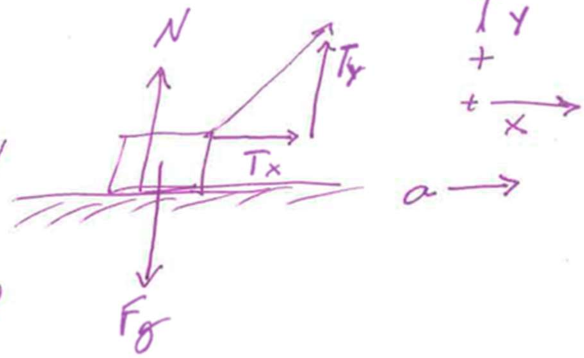
$W = \vec{F} \cdot \overrightarrow{dx}$, We take the x-component of the tension (force) to find that the work I do is *20 N * 5 m = 100 J.*

$\Sigma F_y = 0$

$T_y + N - F_g = 0$

$\Sigma F_y = 0$





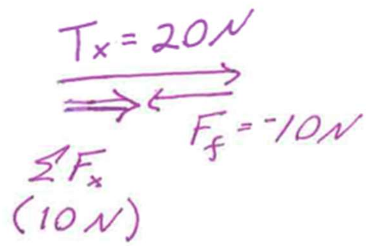
$\Sigma F_x = ma$

$T_x - F_f = ma$

$20N - 10N = ma$

$\frac{10N}{m} = a \approx 1\, m/s^2 \Rightarrow$

$F_f = \mu N$
$= 0.15 \cdot 65.4N \sim 10N$



finding the acceleration requires us to use a dynamics lens because the force cause the acceleration. We do a good FBD as always and identify that the forces in the x direction are the horizontal tension and the friction force. To find the force of friction, we need the normal force. We recognize that we are in equilibrium in the y direction because we are (likely) not accelerating off the surface of the earth. Gravity provides 100 N of force (downward), and the vertical component of tension is 34.6 N upward. In order to be in equilibrium in the y direction, the normal force must be 65.4 N (upward). This yields a friction force of about 10 N in the direction opposite to our motion. Assuming that we are moving forward as I pull the sled, the net force is the sum of the x-component of tension minus the frictional force *20 N – 10 N = 10 N* in the positive direction. This yields an acceleration of the 10 kg sled and girl of 1 m/s^2.