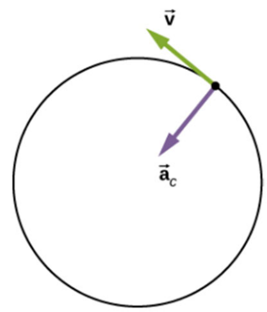
**5.0 Central Attraction:** centripetal acceleration

When we see a body moving in a circular path, we know that there must be force acting on it because without a force, things travel in a straight line. We likely want to look at this through a *dynamics lens* because forces are *causing* this *centripetal* acceleration. From our experiences we can also see that the force is radially inward:

a) Gravitational Force causes centripetal acceleration: Gravity pulls the moon into a circular orbit. The force of gravity on the moon pulls the moon directly inward toward the earth.
b) Tension causes centripetal acceleration: If you attach a rock to the end of a string and spin it around your head in a horizontal circle, the string pulls the rock radially inward.
c) Normal Force causes centripetal acceration: If you roll a marble around the bottom of a cylindrical container, the normal force of the wall on the marble is inward.
d) Friction causes centripetal acceleration: the tires of a car executing a turn bend under the sideways frictional force accelerating the car toward the center of the circle, as in the picture of the car turning to the right.


We know that $\sum \vec{F} = m\vec{a}$, so if the force on the object is inward, we can be sure the resulting acceleration is inward too. At right, you see that the velocity of an object in circular motion is tangential to the circle (and would continue in this direction without some force accelerating it), and the acceleration (caused by a radial force) is radially inward.




We can show that the acceleration of an object in uniform circular motion scales like the square of the velocity and inversely with the radius of the circular path. Consider that at some velocity, *v,* you go around a half turn of radius *R*, and come back with velocity –*v*. Remembering $\vec{a} = \frac{d\vec{v}}{dt}$:

a) If you double your speed the change in velocity (-2*v*) also doubles. However, at twice the speed, you take half the time to make the turn, so the acceleration should increase by a factor of 4, or the square of the speed.
b) If you double the radius of the turn, the change in velocity is still the same (-2*v*), but you take twice as long to do the turn, so the acceleration should be half as much

Thus, we are not surprised that centripetal acceleration is $a_c = \frac{v^2}{R}$.

Forces cause acceleration (dynamics lens) not the other way around. I often hear that *centripetal acceleration* means there's *centripetal force*. There's no such thing as centripetal force, just like there's no such thing as linear force. A force is an interaction between two bodies whereby they exchange momentum. Force can cause linear acceleration or centripetal acceleration. When we see centripetal acceleration (or any other acceleration), we know that there is some force pushing or pulling on the object to cause this acceleration. So, if we see circular motion, we should consider the dynamics lens and look for the force or *net force* toward the center of the circular path.

Exercise 5.0.1: Please first read the previous paragraph.
You see a 10 kg rock in space moving with constant speed of 10 m/s in a circle of radius 20 m. You wonder about the rock, and look at it through different lenses.

a) Do you think there's a force acting on it? Why?

b) Find the acceleration of the rock, including direction of the acceleration.

c) Calculate the force necessary to accelerate this rock.

d) What kind of force is this? – if you say, "it is centripetal force!" I will be sad. I will be pleased if you say, "I have no idea what force is acting on it, because I can't see anything that the rock is interacting with, so I have to look around at what object must be applying a force of_____(put answer from c) on the rock to make it accelerate at______(put answer from b)."

e) Then you see a string attached to my arm as I spin the rock in a circle. What kind of force is it? Find the tension in the string.

f) Then the string breaks – what happens to the rock?
*Please draw a picture.*

g) ….instead of a string, you see a large sphere at the center of the rock's circular path. What kind of force might be acting on the rock now?

h) ….instead, you notice that the 10 kg rock is actually a small 10 kg toy car driving around in a 20 m circle on a flat parking lot at 10 m/s. Now what force is acting on the car? Please find the coefficient of friction necessary to keep the car moving in this circle. Why is this static friction and not dynamic friction?

Exercise 5.0.2:
You're taking a turn with a car or bike on flat pavement and there is a coefficient of static friction of 1.2 between the rubber and the road. To execute a turn of a radius of 10 m,

- what force causes the centripetal acceleration allowing you to make this turn? State your lens; set up the drawings and logic; and reflect on your answers in light of your experiences.

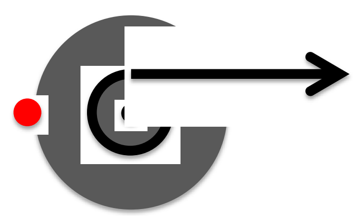
- How fast can you go without "sliding out?"

- Why did we use static friction rather than dynamic friction?

Exercise 5.0.3:

Prove to yourself that we can also write centripetal acceleration as $a_c = \omega^2 R$, and $a_c = \frac{4\pi^2 R}{T^2}$, where $\omega$ is angular velocity, and $T$ is the period of revolution. Astronomers like the second expression because $T$ is the period or time it takes for a full revolution, or length of year (of a planet for instance).

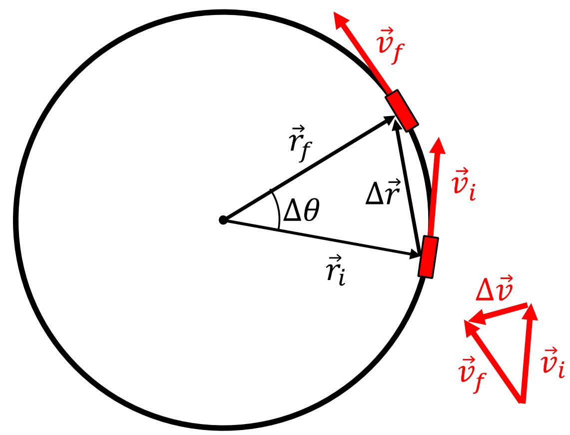
Exercise 5.0.4:
A concrete flywheel of uniform thickness has a mass of 50 kg and a radius of 40 cm. The string is wound around a pulley of radius 16 cm. I pull on the string with a force of 100 N. After some time, the wheel is spinning such that it takes two seconds for a full revolution. At this time, you notice a fly standing on the rim of the wheel at the extreme left (red dot):



a) What's the fly's tangential acceleration? *Include direction*

b) What's the fly's centripetal acceleration? *Include direction*

Exercise 5.0.4:

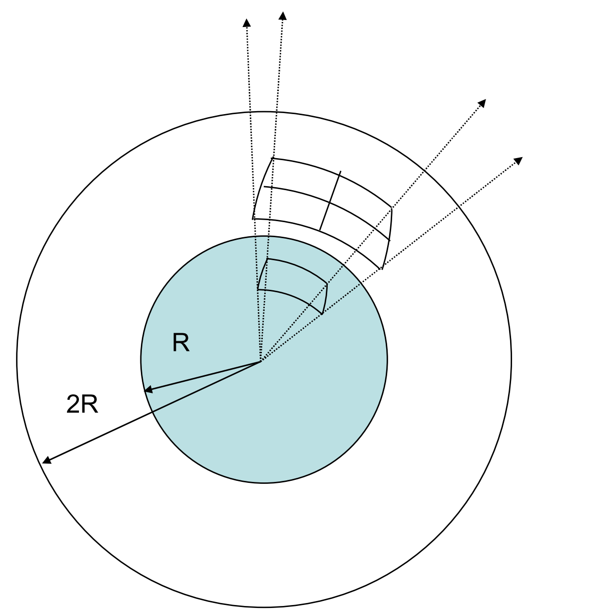



**PROVE IT!** Proving the formula for centripetal acceleration is a classic proof I expect you to be able to do. Please learn it. The drawing at right shows a picture of something moving in circular trajectory. The diagram to the right shows how to find the change in velocity using the velocities at the two points in the circular trajectory.

Please prove to yourself that the two isocelese trianges are similar, and thus their parts are in proportion. You can then start with the definition of acceleration as the rate of change of velocity and express $\Delta\vec{v}$ and $\Delta t$ in terms of radius and speed. In this proof it is also important to take the limit of $\Delta t = \Delta\vec{v} = \Delta\theta \Longrightarrow 0$. Why? Do we want the average acceleration, or the *instantaneous acceleration,* $\vec{a} = \frac{d\vec{v}}{dt}$? The average acceleration over one revolution is zero because the change in velocity is zero. The acceleration, like the velocity is constantly changing, so it's the instantaneous acceleration that is of interest. When we take the limit of short time or small $\Delta\theta$, we see that $\Delta\vec{r}$ (the change in position) becomes the same as the distance traveled along the perimeter of the circle and $\frac{\Delta r}{\Delta t} \Rightarrow v$, the tangential speed. Please carry out this proof with a good drawing.

### 5.1 Ubiquitous Inverse Square Relationship: *It's everywhere*

When things spread out in three dimensions from their source (like light, sound, or shrapnel from a cluster bomb) intensity decreases *proportional* to the inverse of the distance *squared*:
$Intensity \propto \frac{1}{r^2} = r^{-2}$.
Why is that? When things come from a point source, they spread out with *spherical symmetry*, so they are spread out over the surface of a sphere that gets bigger as it expands in time. Because the surface area of a sphere = $4\pi r^2$, the area is proportional to the *square* of the radius. So, if you double your distance to a lightbulb, the light from the bulb is spread out over 4 times the surface area, and the intensity of light at the new location is ¼ as great as before:
If $R \Rightarrow \mathbf{2}R_i, I \Rightarrow \frac{\mathbf{1}}{\mathbf{4}} I_i$.



Light propagates equally in all directions from a point source in the middle. The flux of power that escapes through the smaller sphere must also pass through the larger sphere. Because the same flux is spread over a larger surface at the larger sphere, the intensity of light is weaker at the larger distance.

This may be better seen from the diagram at right, where if light travels through a window of a sphere at distance $R_i$, the same light would need 4 windows to pass through a sphere at distance $2R_i$: 2 windows wide, and 2 windows high.

In the next section, we describe how Cavendish measured the relationship between mass, distance and gravitational attraction. However, even before he experimentally determined this relationship, one could imagine the gravitational force (from a planet) spreading outward into space in all directions. Thus, we might have expected the gravitational force to drop off according to the inverse square relationship.

Exercise 5.1.1: The distance between the centers of the earth and moon is about 385,000 km and the radius of the earth is about 6,400 km.

a) How many earth radii is the distance between the center of the earth and moon?

b) If the moon were to stop moving, estimate the moon's acceleration toward the earth. It may help to know that the acceleration from gravity one earth radius away from the earth's center is 10 m/s$^2$.

c) But the moon *is* moving! In this case, should its acceleration be the same as in "a" above? Why or why not?

d) From your answers above, estimate the speed of the moon in its orbit around the earth.

e) From your answer above, estimate the period of the moon… is it close to a month?

<u>Exercise 5.1.2</u>
In 1930, it was discovered that a beta decay: $n \Rightarrow p^+ + e^-$
didn't conserve energy, momentum or <u>*angular momentum*</u>. Wolfgang Pauli imagined the existence of a new particle, the *neutrino* with the correct angular momentum to conserve angular momentum in this process. We now estimate that 65 billion neutrinos from the sun pass through each square centimeter on earth, *per second*. The earth is about 150 billion meters from the sun and Venus is about 108 billion, or about 0.72 (or $\sim\frac{1}{\sqrt{2}}$) the earth-solar distance.

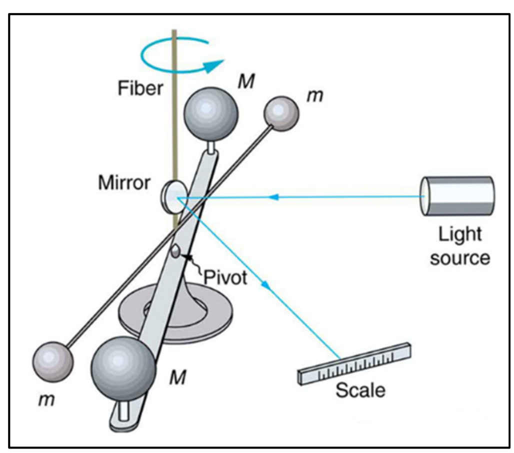
- How fast does the sun produce neutrinos?
- How many pass through you during a one hour class?
- How about if you were on Venus?

It's interesting to note that neutrinos interact with matter very very weakly making them very difficult to detect. In fact, the flux of solar neutrinos is negligibly attenuated by the earth, so the flux coming out of the earth at night is about the same as that entering the earth on the other side facing the sun. This was astutely noted by John Updike in 1960:

*Neutrinos they are very small.*
*They have no charge and have no mass*
*And do not interact at all.*
*The earth is just a silly ball*
*To them, through which they simply pass,*
*Like dustmaids down a drafty hall*
*Or photons through a sheet of glass.*
*They snub the most exquisite gas,*
*Ignore the most substantial wall,*
*Cold-shoulder steel and sounding brass,*
*Insult the stallion in his stall,*
*And, scorning barriers of class,*
*Infiltrate you and me! Like tall*
*And painless guillotines, they fall*
*Down through our heads into the grass.*
*At night, they enter at Nepal*
*And pierce the lover and his lass*
*From underneath the bed – you call*
*It wonderful; I call it crass.*

The poem, *Cosmic Gall* was published before the 1998 discovery that neutrinos actually do have mass.

**5.2 Universal Gravity**




Gravity attracts any two bodies with masses *m* and *M*:
$F_{mM} = \frac{mM}{r^2}G$, where *r* is the distance between the centers of the two bodies, and
$G = 6.67\ x\ 10^{-11}\ \frac{\mathrm{Nm^2}}{\mathrm{kg^2}}$, ⟵ m for meters
Cavendish measured gravitational force with an apparatus like that at right.[1] The two small masses *m* are attracted to the larger masses, *M*, causing a rotational deflection of the hanging barbell.

The force of gravity is very weak compared to the force of electric charges. If two electrons are close to each other, the repulsion from the electric charges will be greater than the gravitational attraction by more than a factor of $10^{43}$. The only way we are able to feel the

[1] From OpenStax.org: https://cnx.org/contents/jQSmhtXo@14.60:OMxfrjQL@3/Newtons-Universal-Law-of-Gravi

earth's gravity is because the earth is very massive, and both the earth and our bodies have almost no charge since we have roughly equal numbers of protons and electrons. The force of gravity being so weak made it difficult to measure the gravitational constant. Cavendish's torsional pendulum was very sensitive to the tiny torque produced by the weak gravitational attraction between m and *M*. Additionally, the light reflecting from the mirror allowed him to measure very small rotational displacements.

<u>Exercise 5.2.1:</u>
We know about gravitational force on the earth's surface, and that the distance from the North Pole to the equator along the earth's surface is 10 million meters (the original definition of the meter). With this information, please calculate the radius of the earth and the mass of the earth. Check your answer against a known value.

Before the Cavendish Experiment, people likely expected the formula to have the general form of the product of the two masses divided by the square of the separation of the centers:

- Our last chapter discussed the inverse square relationship that we expect for field interactions (that is, light, electric fields, gravitational fields, etc.)
- We already knew that the force of gravity on an object is proportional to the object's mass, <u>*AND*</u> that a force is an interaction between <u>two bodies affecting each with equal strength in opposite directions</u>. Thus, if we double the mass of one of two gravitationally attracting bodies, the force of gravity on <u>both</u> the bodies must double.

**The Satellite Equation**: In the vacuum of space, the only force acting on a body is often gravity. For circular orbits, centripetal acceleration results from this force. Thus, for circular orbits, the Satellite Equation can be written: $F_G = ma_c$

<u>Exercise 5.2.2</u>, Lower Earth Orbit (LEO):
Above an altitude of 100 miles (160 km), the atmosphere is thin enough to allow near frictionless orbiting.

- Make an argument that at this elevation, the acceleration of a satellite is reasonably close to that at sea level, *g*.
- What is the speed, and orbital period, *T*, of a satellite in LEO?
- If you were in LEO, about how many times a day would you expect to see the sun rise?
  Read up on Lower Earth Orbit by looking it up on Wikipedia (or somewhere else).

Exercise 5.2.3, Geosynchronous Orbit (GSO): Communication satellites are used to bounce signals around the curve of the earth. It is nice for them to stay over the same place on the earth by orbiting the earth in a single day.

- LEO orbits have a period of way less than a day (see above exercise). Please explain why we might think that as we get further from the earth the orbital period would increase.

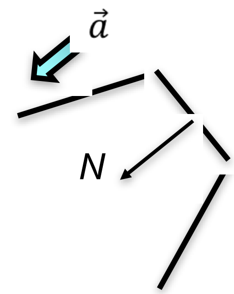
- With a full drawing and reasoning, calculate how far from the earth's center GSO is. Please put your answer in terms of the number of earth radii. Before you calculate it, consider:
  - What lens will do you use?
  - Provide a good lens discussion and set up the problem.
  - Does your answer make sense to you?
  - Should GSO be closer or further than the moon? How can you be sure?
- Please calculate the radius of a geosynchronous orbit.

- If you were in GSO and stopped moving, you would start falling toward the earth. What would your acceleration be?

Force is a vector! We know that gravity is an attractive force, so the force is always in the direction of $-\hat{r}$. The vector notation of the force of gravity then becomes:
$\vec{F}_{mM} = -\frac{mM}{r^2}G\hat{r}$.

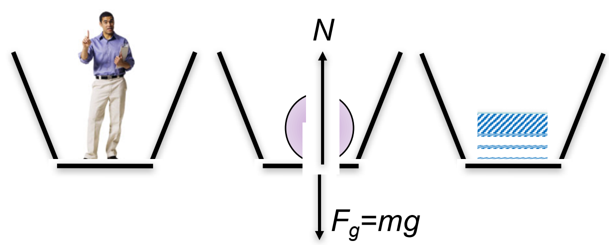
**5.3 Loop-the-Loop:** *Circular motion in the vertical plane*

When you see something moving in a circle, it's a good bet that dynamics is involved. Make a free body diagram and note acceleration is towards the center of the circle. We will address circular motion when there is more than one force, such as in a roller coaster loop, there is gravity and the normal force of the seat acting on your body!




<u>A review from section 2.5 (just this page)</u>:
If we were in outer space, or in free fall inside the space station as it orbits the earth, there is no apparent force of gravity. How would we keep water inside a bucket, so that the water is touching the bottom of the bucket? "Touching" the bottom of the bucket means that there is a nonzero normal force; that is $N > 0$. Given that this is the only force, it would mean that the bucket would be accelerating in the same direction. So, you could imagine accelerating the bucket back and forth or up and down while rotating it to maintain a positive normal force.

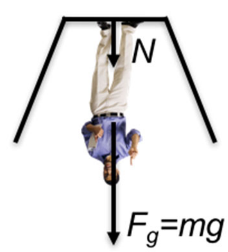
Please recall the elevator problem noting that $\sum \vec{F} = m\vec{a}$. Consider keeping a man, a ball, or water in a bucket as shown at right.



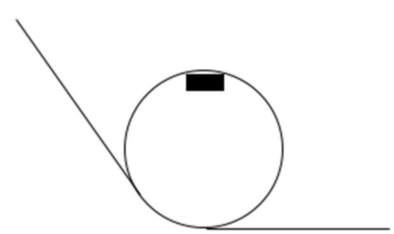

<u>Exercise 5.3.1</u>:
If the object is kept in the bucket, what do we know? Identify all correct statements. Answers at chapter end.

a) $N > F_g$.
b) $F_g > N > 0$.
c) $N > 0$.
d) acceleration must be upward or zero only.
e) acceleration can be zero or any value upward or downward.
f) acceleration can cannot be downward with a magnitude more than gravity.

<u>Exercise 5.3.2</u>:
How about if we see someone in a bucket upside down, what do we know? Identify all correct statements. Answers at chapter end.




a) $N > F_g$.
b) $F_g > N > 0$.
c) $N > 0$.
d) acceleration must be downward with a magnitude more than gravity.
e) acceleration can be any value, but must be downward.
f) acceleration can be zero or any value upward or downward.
g) acceleration can cannot be downward with a magnitude more than gravity.



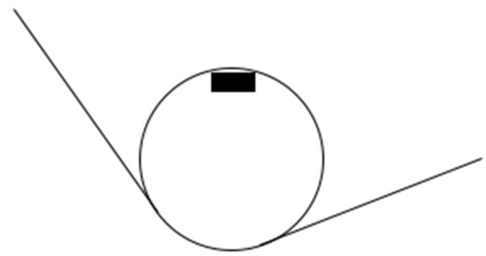
When we see something moving in a circle, we only know that it is accelerating radially inward. We then look for the forces that could provide that acceleration according to $\sum \vec{F} = m\vec{a}$. So, for instance with a "Loop-the-Loop" carnival ride, the forces include gravity and the normal force that cause centripetal acceleration inwards. So, at the top the centripetal acceleration is downward. If we want the cart to stay in contact with the track, we know that the normal force must be downward. What does this say about the centripetal acceleration resulting from gravity and the normal force?

Exercise 5.3.3:
Please make a good free body diagram of the cart at the top of the loop and the bottom of the loop. Please identify what you know for both the *acceleration,* and the *normal force*:

- at the top of the loop                                  at the bottom of the loop

be sure to support your answer with a diagram and lens identification.

Exercise 5.3.4:
You go on a $R = 10$ m, loop-the-loop ride where the cart is let go on a low friction track and is pulled downhill by gravity. You have to choose how high to start the cart. Say you have a mass of 70 kg, like your instructor and you are sitting on a scale that reads in kg. Don't use this drawing… please make your own.



a) If you start from a vertical height of 40 m, what does the scale under you read as you are at the top of the loop? What does it read at the bottom of the loop? Is this a good ride for pregnant women? How does it feel as you round the bottom of the loop?

b) What would happen if you decide to start the cart at the same height as the top of the loop? Why would this happen?

c) Please find the minimum vertical height, above the ground that you must start the ride to stay on the track at the top.

The low friction, circular loop-the-loop ride above has a higher speed at the bottom than at the top. Thus, the centripetal acceleration in the circular path is also greater at the bottom. Additionally, please show that because of the way the track faces, in order to keep the cart on the track at the top, there must be a very great normal force at the bottom. This could make for a dangerous carnival ride! Is there some way we could increase the centripetal acceleration at the top (so the track stays connected to the cart), and lower the centripetal acceleration at the bottom (in order to not smoosh the people)?

Exercise 5.3.5:
Look at the carnival ride at right. Notice that the radius of curvature at the top is not the same as the radius of curvature at the bottom. What effect does this have on the centripetal acceleration at the top and at the bottom? Why would they build it this way? Please support your answers with a good free-body diagram, and identify the lenses you use.

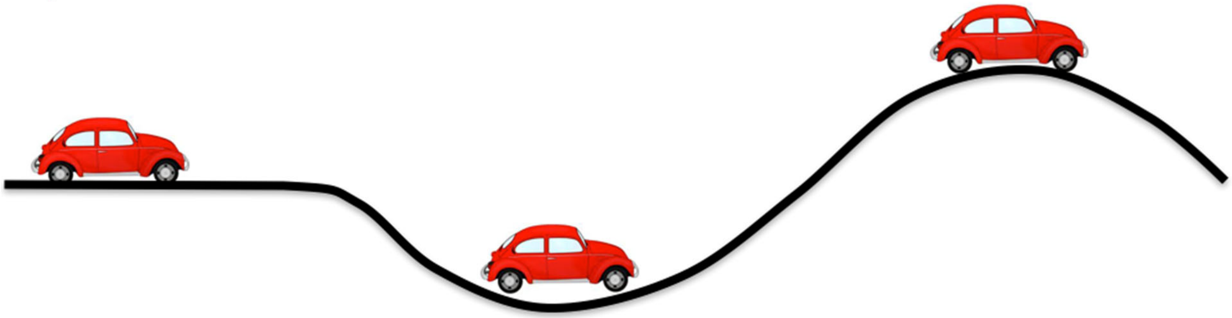
Exercise 5.3.6:
Below, you see 3 pictures of a car you are driving. You are sitting on a common "bathroom scale" that is situated between your body and the seat.

a) Do you "feel" the same at the three different places? If not, how do you feel differently?

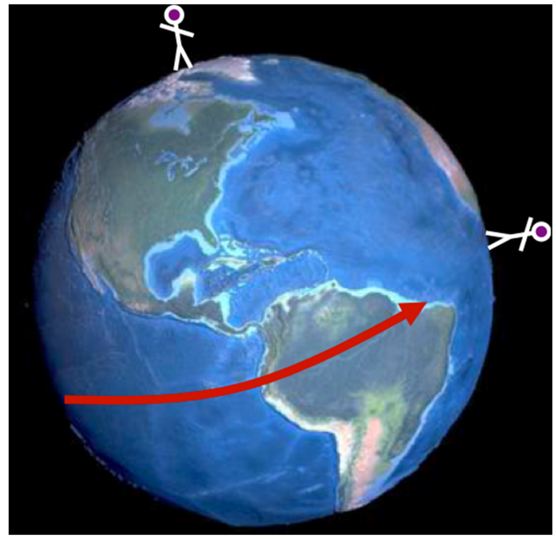
b) Does the scale read the same at all three locations? If so how do you know? If not, where is the reading the highest and where is the reading the lowest?

c) Please back up your answers above with a good free body diagram and indication of direction of acceleration.

d) In each of three places, how would the reading on the scale compare to the force of gravity on your body?



Exercise 5.3.7:
*Do you weigh more at the North Pole?*
You stand on a scale at the north pole and then on the same scale at the equator to see where the scale reads higher. Assume that the earth is a perfect rotating sphere. You find that at the equator, the scale reads:



a) More because the normal force compensates for both gravity *and* centripetal acceleration.
b) Less because the normal force at the equator is not enough to keep you in equilibrium.
c) The same because the normal force is always the same as gravity.
d) The same because you are in equilibrium in both places
e) None of these.
f) Not enough information is given

If you get confused… as with any mechanics problem, please draw a picture and do a lens analysis. There is a story that gold was mined in Alaska, weighed, and sent to Fort Knox in Washington DC where they weighed it again. What did they think in DC when they weighed the gold they'd just purchased?
*Please don't read these answers until you have made a free body diagram and discussed the answers yourself. Exercise 1 (c,f); Exercise 2 (c,d)*

## 5.4 Gravitational Potential Energy in Outer Space

We learned in section 1.7 and 2.7:
$W = \Delta E = F\Delta x$, and
$dW = dE = Fdx$. So, if you push a car on flat ground with a force of 200 N for a distance of 10 m, you do 2000 J of work, which increases the car's kinetic energy by 2000 J. If you change the magnitude (strength) of the force on the car half way through, you'd have to calculate the work done in each section and add it to calculate your total work, or the total change in energy of the car.

$$W = \sum_i W_i = \sum_i F_i \Delta x = F_1 \Delta x_1 + F_2 \Delta x_2$$

What if the change is not discreet, actually changes smoothly? Then the sum becomes an integral of force over the distance:

$$W = \int dW = \int_{x_o}^{x_f} F dx$$

Apply to gravitational potential energy:
When you raise an object some distance *dy*, the work you do on it is equal to the gain of gravitational potential energy, $\Delta E_G$:
$W = F_g \Delta y = mg\Delta y = mgy_f - mgy_o$

This works as long as the force of gravity is a constant, $mg$. What do we do for changes in elevation so great that the force of gravity changes? The force of gravity drops off as the inverse square of the distance to the *center of a planet*, as described in sections 5.1 and 5.2:
$F_{mM} = \frac{mM}{r^2} G.$

The work and $\Delta E_G$ is found by integrating this force over the change in height, $dH = dr$.

$E_G = -\frac{mM}{r}G$, so

$\Delta E_G = \frac{mM}{r_i}G - \frac{mM}{r_f}G = mMG\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$

Exercise 5.4.1:

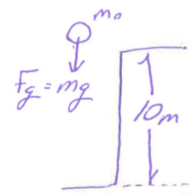
With a good drawing, consider moving a mass from the earth's surface, at $r = r_o$, to some greater elevation at $r = r_f$. Integrate the force of gravity over the elevation change to derive the formula for $\Delta E_G$ above. Be careful of signs and consider whether you are doing positive work.

Exercise 5.4.2:

Let's use the different equations for gravitational potential energy

- You drop a rock from a 10 m cliff. Why can we use $\Delta E_G = mg\Delta y$ for to find the final speed? What lens do you use to find the final speed? Please find the final speed when the rock hits the ground.
- You drop a rock from 6.4 million meters (about one earth radius) above the earth's surface. Why is it no longer reasonable to use $\Delta E_G = mg\Delta y$? Would the correct speed be less than or greater that that calculated with $\Delta E_G = mg\Delta y$? Why? Use the correct formula for $\Delta E_G$ to calculate the speed of the rock when it hits the ground.

Because the $F_g$ doesn't really change in 10m (because $10m \ll r_e$), we can use $\Delta E_g = mg\Delta h$




This is an energy lens problem because $E_g \Rightarrow E_K$ $\quad \Delta E_K = -\Delta E_g$

$$E_{K_f} = -\Delta E_g$$

$$\frac{1}{2}\cancel{m}V_f^2 = \cancel{m}\cancel{g}(-\Delta h)$$

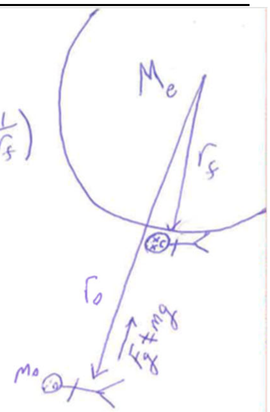
$$V_f = \sqrt{2g\Delta h} = \sqrt{2\cdot 10\,\tfrac{m}{s^2}\cdot 10m} = \sqrt{2}\cdot 10\,\tfrac{m}{s}$$

$$\approx 14.4\,\tfrac{m}{s}$$

makes sense, fall 10m, $V_f \approx 30\,\frac{mi}{hr}$

---

Because $\Delta r$ is comparable to $r_e$, $F_g$ isn't constant through the fall + we need to use $\Delta E_g = m_o M_e G\left(\frac{1}{r_o} - \frac{1}{r_f}\right)$




if $V_o = 0$ then $\Delta E_K = -\Delta E_G$

$$E_{K_f} = \frac{1}{2}m_o V_f^2 = -m_o M_e G\left(\frac{1}{r_o} - \frac{1}{r_f}\right)$$

$$\left(\frac{1}{2r_e} - \frac{1}{r_e}\right)$$

$$\frac{1}{\cancel{2}}\cancel{m_o}V_f^2 = -\cancel{m_o}M_e G\left(-\frac{1}{\cancel{2}r_e}\right)$$

$$V_f = \left(\frac{M_e G}{r_e}\right)^{\frac{1}{2}} \approx \left(\frac{6\times10^{24}\,\cancel{kg}\;6.7\times10^{-11}\,Nm^{\cancel{2}}}{6.4\times10^{6}\,\cancel{m}\;kg^{\cancel{2}}}\right)^{\frac{1}{2}}$$

$$\approx \left(6\times10^{7}\,\frac{m^2}{s^2}\right)^{\frac{1}{2}} \approx \left(64\times10^{6}\,\frac{m^2}{s^2}\right)^{\frac{1}{2}}$$

$$\approx 8\times10^{3}\,\tfrac{m}{s}$$

$$\frac{Nm}{kg} = \frac{\cancel{kg}\,\frac{m}{s^2}\,m}{\cancel{kg}} = \frac{m^2}{s^2}$$

Exercise 5.4.3:

Escape speed is the speed you need on a planet's surface to take you "infinitely far away." This is *not* infinite because as you get far from the planet, the force of gravity gets weaker.

a) Calculate the escape speed from the earth's surface: how fast do you need to throw a rock in order for it to not to come back to earth?

b) 14 km/s is faster that the escape speed from the earth. So, if you could throw a rock at 14 km/s, it would still have kinetic energy when it got very far from the earth. What would the rock's speed be infinitely far from the earth?

c) Note: You really couldn't get infinitely far from the earth with this speed because you would also need to escape from the solar system and then the galaxy requiring much more kinetic energy than just escaping from the earth alone. Consider for a moment how we would we consider the sun's gravity?

Is there an absolute value of $E_G$? It is arbitrary where we assign $E_G = 0$ because the physical behavior is determined from $\Delta E_G$. For instance, if you take a physical system like a mass on a spring and increase its elevation by 1 m, it still behaves the same. On earth, we can assign $E_G = 0$ to sea level, but it can also be the desk top in your laboratory. But what if we want to compare the potential energy of the earth's surface with the surface of Mars? For this, we assign $E_G = 0$ at $r = \infty$, when you are infinitely far from any massive body. This can be considered a "free" mass. By integrating back from $r = \infty$ to some distance $r_f$, to a planet of mass $M$, we have:

$E_G(r) = -\frac{\mathrm{m}M}{r}G.$

Our potential energy is *negative* everywhere! Yes, this would be true, because we do *negative work* lowering a mass down to a planet from $r = \infty$. The force we apply to the rock is in the opposite direction we are moving. We say that on a planet, we are "stuck in a gravitational well". And we are! Just jump up off the earth's surface toward outer space and see what happens.

Exercise 5.4.4:

There are two planets made out of the same material, except one has twice the radius of the other. That is $r_A = 2r_B$. I discover that the escape speed of the larger planet (planet A) is 100 m/s. *Clearly Explain Both Answers Below.*

a) What is the escape speed of planet B?

b) If I am catapulted from the surface of planet B with a speed of 100 m/s, will I escape the planet? If not, how high will I get? If I do, what will be my speed when I get to infinity?

c) If this planet is made of the same material as earth. How do you think its size would compare to earth? Please estimate the planet's size.