Part 3

NEWTON'S LAWS

Newton's Laws

Force Diagrams

Setting Up Newton's Law Problems

Force Problems

Isaac Newton was born in 1643, the same year that Galileo died - as if it were somehow necessary to have at least one person of that caliber living at any given time.

David Goodstein

Newton writes F=ma and thereby captures a whole universe of mechanical interactions. This way of expressing ideas contrasts sharply with the method of the humanities. Shakespeare cannot condense the totality of his thoughts and feelings about Lear into a sentence like "Lear is mad." Instead he must write the play in all its redundancy, ambiguity, vagueness, verbosity, and obscurity. But the comparison is misleading. Hidden behind the equation F=ma are the definitions of the symbols, the philosophical problems of their interpretation, the historical antecedents of the theory and its complicated, imprecise and ambiguous experimental tests, its realms of applicability and limitations, its practical consequences, its equivalent formulations - in short, its meaning. The physicist who appreciates the beautiful conciseness of the equation is aware of the vast messy world of physics in the background without which the four little symbols would be senseless. He is cheating when he pretends that F=ma tells the whole story, just as the scholar is cheating when he substitutes a synopsis of Lear for the play.

Hans C. von Baeyer
Rainbows, Snowflakes, and Quarks

If you push something, it speeds up. If you press against something, you feel it press back against you. The more massive something is, the stronger its gravitational pull.

Brian Greene

NEWTON'S LAWS

If physics is about the motion of objects, the description of motion - the mathematics used to determine the position of some object as a function of time - is a necessary component of the subject. But physics is not just the description of how things move, but rather the articulation of the reasons why objects move as they do. It is ultimately about the interactions between the bits of matter that relate to their motions. The principles that govern the motion of objects is what is ultimately important. Isaac Newton (in the late 1600s) related the motions of objects to the forces that cause those motions in his three laws of motion. And those ideas subsequently led to the universal law of gravitation the explanation of the planetary orbits and Kepler's laws, and the complete description of the motions of all objects that are a part of our normal experience. Newton's laws describe our world.

Newton's 1st Law: The Law of Inertia

In the absence of a net force on an object, the object either remains at rest or moves in a straight line at constant speed.

The first law just identifies that changes of motion occur because of forces - and so the state of motion remains unchanged either if no forces act or if all the forces that do act are balanced in such a way to add to zero. This is the basis of all equilibrium problems. The *inertia* of an object is its tendency to remain in its current state of motion. The object's mass is a measure of its inertia.

Newton's 2nd Law: The Law of Motion

The result of a net force (or unbalanced forces) on an object is an acceleration in the direction of the net force. The acceleration will be numerically equal to the net force that acts on the object and inversely proportional to the object's mass.

$$\vec{\mathbf{F}}_{\mathbf{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad \text{or} \qquad \vec{\mathbf{a}} = \frac{1}{m}\vec{\mathbf{F}}_{\mathbf{net}}$$

This is the central idea of classical mechanics. If you can determine the forces that act on an object – both magnitudes and directions, you can also determine the resulting acceleration, from which the velocity and position as functions of time can be determined. That is, $\vec{\mathbf{F}}_{\mathbf{net}} = m\vec{\mathbf{a}}$ lets you solve for the motion of the mass. Notice that the second law "contains" the first: If the net force is zero, so is the acceleration - and the vector velocity of the object is then a constant (ie, either the object remains at rest or it moves in a line at constant speed). Also notice that both force and acceleration are vectors.

Newton's 3rd Law: The Law of Interaction

All forces are the result of an interaction between two objects. If an object exerts a force on some other object, there is an equal and opposite force being exerted on the first object by the second.

This may be the most important of the three laws. It says that *all* forces are caused by some other object. Understanding the third law is essential to being able to solve problems using the second. In order to correctly set up any force problem, it is necessary to identify the forces that act on each object. It can be summarized by the equation $\vec{\mathbf{F}}_{1\to2} = -\vec{\mathbf{F}}_{2\to1}$ - meaning the force exerted by object 1 on object 2 is equal in magnitude but opposite in direction to that exerted on object 2 by object 1. This important idea will lead to the conservation of linear momentum and the conservation of energy.

Although these three laws, articulated by Isaac Newton to explain the motions of all objects, they are not independent of a *fourth* law – and the law that he is also known for, the law of Universal Gravitation. This idea was developed in concert with his second and third laws to explain the motion of the moon about the earth – and by analogy, the interactions of all objects through their mutual gravitational attraction.

Newton's Universal Law of Gravitation

Any two objects have a mutual gravitational attraction that depends only on their masses and the separation between them.

The magnitude of the gravitational force is given by : $F_g = G\dfrac{m_1 m_2}{r^2}$

Newton considered the question of why the moon orbits - that is, it must be in a constant state of freefall about the earth the same as an apple which falls near its surface. The third law states that the earth and moon must have equal forces - each due to the other, and Newton reasoned that each force must depend on the *product* of the masses. Newton deduced that the "force law" would have to be of the form $F_g=Gm_1m_2/r^2$ where G is some *universal* constant by comparing the accelerations of a falling apple and the centripetal acceleration ($a_c=v^2/r$) of the "falling" moon. He then subsequently derived Kepler's laws of planetary motion - showing that they are in elliptical orbits.

This "law" is important because (in addition to predicting the planetary orbits) it explains why all objects on earth have the "same" acceleration in freefall if there are no other forces acting. That is, if M is the mass of the earth and R its radius, then the force on an object of mass m at the earth's surface is given by

$$F_g = G\frac{Mm}{R^2} = m\left(G\frac{M}{R^2}\right) \qquad \text{where } G = 6.67\text{x}10^{-11}\ \text{Nm}^2/\text{kg}^2.$$

That is, since $F = ma$, then the acceleration is $a=GM/R^2$ which will be nearly the same at all points on the earth's surface – differing only because of the variation in the radius of the earth.

The Universal Gravitational Constant G was determined experimentally by Henry Cavendish in the mid-eighteenth century, about a century after Newton articulated the law of gravity. We call the freefall acceleration near Earth's surface g which has the measured value of approximately 9.81 m/s^2. So the force on an object due to gravity is given by $F_g = mg$ - a quantity most authors call *weight* , but should be called *force due to gravity*.

Now we can understand the significance of Newton's laws and how a believed idea (the constant acceleration under the influence of gravity) is a result, not an assumption (or a given). Also, the value of "g" is actually not a universal constant but depends on R, which is not the same at all points on our non-spherical earth. Finally, measuring g , and having an approximate value for R, (known since the time of Eratosthenes in 300 B.C.) and the value of the gravitational constant G (from Cavendish), the mass of the earth can be calculated!

COMMENTS ON NEWTON'S LAWS OF MOTION

- Forces are never the cause of motion, but rather the cause of changes in motion.

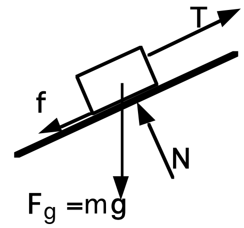
- Forces are always the result of an interaction. Forces on objects are always the result of some other object exerting the force. That point is not self-evident to most beginning students and is the very core of Newton's laws - and ultimately leads to both momentum and energy conservation.

- Newton's 2nd law is for the resultant force or the net force. It states that the consequence of all the forces acting on an object is that it accelerates in the direction of the vector sum of all the forces. The equation $\mathbf{F}=m\mathbf{a}$ does not define force, but rather relates the change in motion of an object to all of the forces that act on it. That is, "ma" is not just some additional force that is needed to balance a force equation - but rather the response of an object to all of the forces that act on it. That point must be understood clearly.

- *Inertia* is the property of an object which describes its tendency to remain at rest or in uniform motion. The *mass* of the object is a measure of its inertia. The concept of inertia will resurface in the discussion of rotational motion with the quantity *moment of inertia* as a measure of an objects rotational inertia.

- It is in the discussion of "weight" that I depart from nearly all introductory texts. Any place where the word "weight" is used, "the force due to gravity" is what is meant. The difference is not just semantic. "Weight" brings to mind a property of an object, whereas "force due to gravity" is something that acts *on* the object. I would prefer saying that the mass *m* is the property that is acted on by gravity - resulting in a force $F_g=mg$. The resulting acceleration if that is the only force that acts is $a_g = g$ which then equals 9.81 m/s^2 when the object is in freefall near the surface of the earth.

- In general, forces are imposed on an object either by direct contact or by "action at a distance" - ie, via the effect of some *field*. The only force that we will consider in this course that is not by direct contact is the gravitational force. So <u>all</u> forces that we will consider are either gravitational in nature or are the result of direct contact with the object whose motion is being described. (It should be mentioned that even what we are calling "direct contact" are, on a microscopic scale, due to the forces of interaction between atoms - which are ultimately due to electric fields associated with molecular forces.)



SOME COMMON FORCES

Force Due to Gravity

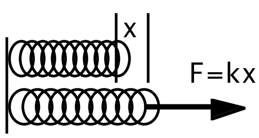
As stated above, since all objects have mass, the motions of all objects on Earth are, in general, affected by the gravitational force. So the force due to gravity must be included in the discussion of the forces that act on any object. In constructing force diagrams to assist in setting up problems, it is always useful to draw the gravitational force first - and letting all other forces be shown in their proper proportion to that force. That will give you a good visual sense of what forces dominate in the problem. The figure shows a block being pulled up an incline by the tension in a rope or string. The relative magnitudes of all the forces involved are drawn approximately correctly.

I suggest avoiding the use of the word "weight" - as well as the use of the symbol "w" for that force - when dealing with the gravitational force and use "force due to gravity" instead. Always let the force gravity exerts on an object of mass *m* be written $F_g=mg$. That will always remind you that gravity is a force that acts *on* the object being described, and not a property of the object itself.

Contact Forces

Contact forces can be associated with *tensions* in strings, ropes, springs, etc., *compressions* in rigid objects like rods, tables, and springs, *frictional* forces, and forces associated with *collisions* (either two objects themselves collide - which cause compressional forces - or there is air resistance which is just the effect of all the collisions with the air molecules in the path of the moving object).

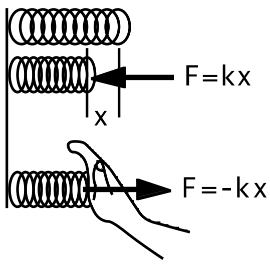
Spring Forces - Hooke's Law



The discussion of springs is important because it relates to all forces associated with tensions and compressions. When a spring is extended or compressed, it tends to resist that extension or compression by its own internal forces. The force required to extend or compress an ideal spring is proportional to the amount that it is distorted from its equilibrium - or *rest* - length. That obser-vation is called Hooke's Law (after Robert Hooke, a contemporary of Isaac Newton's in seventeenth century London). That is, the force required to stretch or compress a spring is given by $F = kx$, where x represents the amount of the extension or compression. And correspondingly, the spring exerts a reaction force equal and opposite to that:

$$F_{spring} = -kx$$

The negative sign simply implies that the spring force is opposite to the force that stretches or compresses the spring.

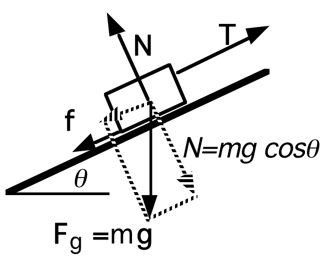


Similarly, when a spring is compressed by an applied force, it will exert an equal and opposite force against the object that is compressing it. Both tension and compression forces can thus simply be thought of as spring forces. When you pull on a rope, it does not stretch very much, but it does stretch slightly and hence exerts a reaction force on you equal to the force you apply to it. And when a solid is compressed, it distorts slightly and responds with an equal and opposite reaction force against the object that compressed it.

All tensions and compressions are thus similar to spring forces. A force is applied to stretch a string or rope or spring and the tension created then exerts a force on whatever object it is attached to. Additionally it exerts an equal and opposite force on whatever pulls on it - a consequence of Newton's third law. Similarly, when a force compresses a spring or rod or any solid object, an identical force is exerted against whatever resists the compressed object and it exerts an equal and opposite reaction force against whatever compresses it (Newton's third law).

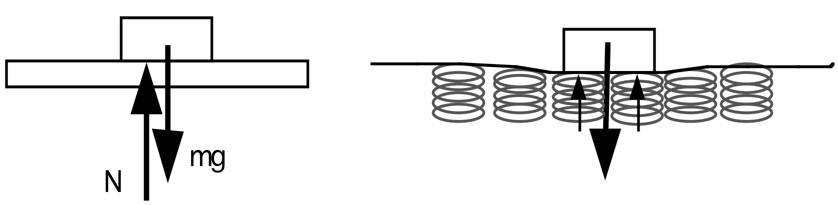
Normal Force

The concept of *normal force* is real obscure for many students - and it needn't be. The normal force that acts between two objects in contact is just the compression force between them. The word "normal" here means "perpendicular". That is, the normal force between two objects in contact is the force perpendicular to the surfaces in contact and is just the force that each object exerts on the other (Newton's third law) associated with the two surfaces compressing each other. And exactly the same normal force acts on each object but in opposite directions.



In the incline plane problem already mentioned, the normal force eexerted by the block against the incline is just $mg \cos \theta$, the component of mg that is perpendicular to the incline (whose angle with respect to the horizontal is θ). So by Newton's third law, the incline exerts an equal and opposite force against the block. That force is shown as **N** and is drawn from the center of the block away from the compressed surface of the incline. The magnitude of the normal force on the block is $mg \cos \theta$,.

Confusion about the normal force can be avoided, I think, by dealing carefully with Newton's third law, and why spring compression will yield a restoring force exactly equal to the force causing the compression. The normal force at a surface can then be thought of as a distributed spring problem. Ie, any surface distorts due to a force until the restoring force equals the compressing force. Similarly, tension in a string or compression in a rod can be dealt with using Newton's third law. Understanding these ideas clearly early on will save a lot of grief later.

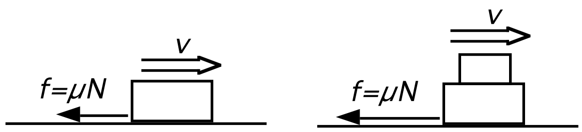


Frictional Forces

Frictional forces are the contact forces between surfaces and are *parallel* to the surfaces (as opposed to perpendicular as are the normal forces). The frictional forces are due to the molecular forces of interaction between the surfaces.

Kinetic Friction

If two surfaces are in contact, but in relative motion - that is, if the surfaces slip on one another, the frictional force is called *kinetic friction*. The force depends on the nature of the two surfaces, the materials involved, and the normal force that compresses the two surfaces together.



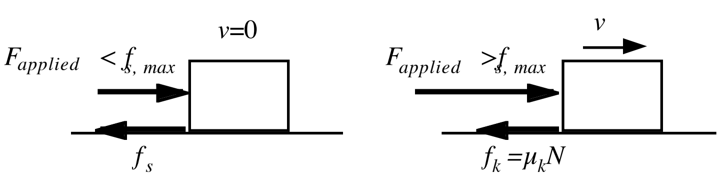
The force due to kinetic friction has a magnitude equal to $f = \mu_k N$, where μ_k is called the *kinetic coefficient of friction* and N is the magnitude of the normal force. And the kinetic friction force always has a direction opposing the relative motion of the two surfaces (and acts on each object - according to Newton's third law). Increase the normal force, and the frictional force increases as well. The direction on each object is always opposite that of the relative velocity - but, in general, does not depend on the speed of the block.

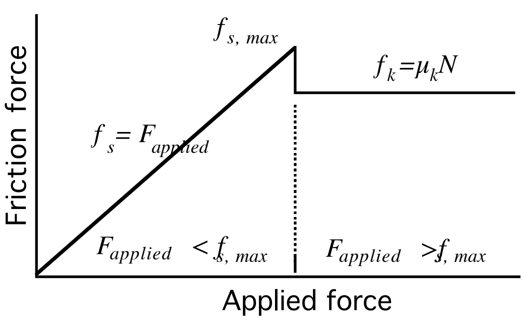
Static Friction

Static friction is similar to kinetic friction, except that it exists only when the surfaces are *not* slipping on each other. That is, the molecular forces between the surfaces in static contact offer resistance to the relative motion. The force of static friction always opposes a *force* that would otherwise make the surfaces slip on each other. If there is no shear force trying to make one object slide on the other, the static frictional force is also zero. Static friction will oppose such a shear force up to some maximum value that depends on the surfaces and on the normal force pressing the two surfaces together. That is,

$$f_s \leq f_{max} = \mu_s N$$

where μ_s is the *static coefficient of friction* .





When that maximum static frictional force is exceeded by the shearing force, the surfaces slip on each other and the static frictional force no longer exists - but the kinetic friction associated with surfaces slipping on each other then appears. Typically, the static friction coefficient is greater than the kinetic friction coefficient, *i.e.*, $\mu_s > \mu_k$. So if you push horizontally against a block which is initially at rest on a horizontal surface, the force you apply does not move the block until the static friction is overcome. At that point, the block slips and the force required to just maintain a constant speed motion of the block relative to the surface is less than the force required to initiate the slipping. That can be represented graphically by showing the friction force as a function of the applied force. The static friction force exactly opposes the applied force until it reaches it's maximum value $\mu_s N$, then the friction if purely kinetic and is equal to $\mu_k N$.

FORCE DIAGRAMS

Force diagrams are very valuable tools to help solving force – acceleration problems. But to make them valuable you need to construct the diagram so that it is clear what all the forces are that act *on* the object you are trying to describe the motion of, and to construct those force vectors large enough in the diagram to clearly represent the relative magnitude of those forces (*i.e.,* make the force vectors proportional to the forces that are acting. That is most easily done if the force vectors are attached to the object itself (and preferably at the point of application of the force – although that is not critical until rotational motions are being considered).

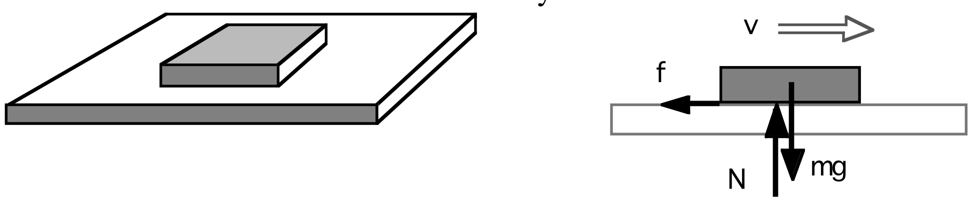
- Show all forces that act on the object – but *not* the forces that that object exerts on other objects.

- Construct all force vectors proportional to the actual forces that act on the object. Constructing the gravitational force *first* will make it easier to properly scale the other forces.

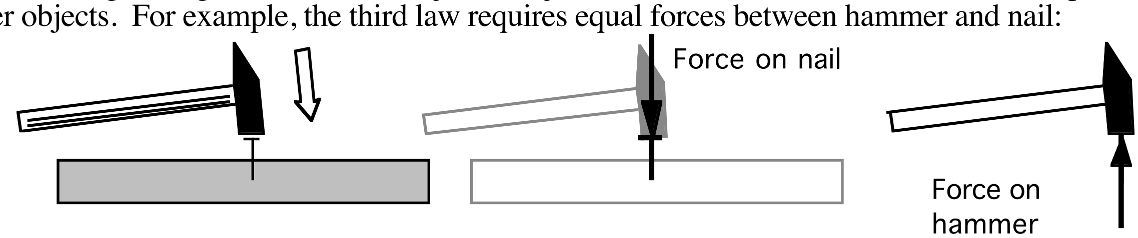
- Never show a force vector disconnected from the object that is subject to the force. And avoid showing other vectors in the figure that are not forces – or if a velocity or acceleration vector *is* included, separate that vector from the object so that it does not appear to be another force.

- Never show a force without being clear what is causing the force.

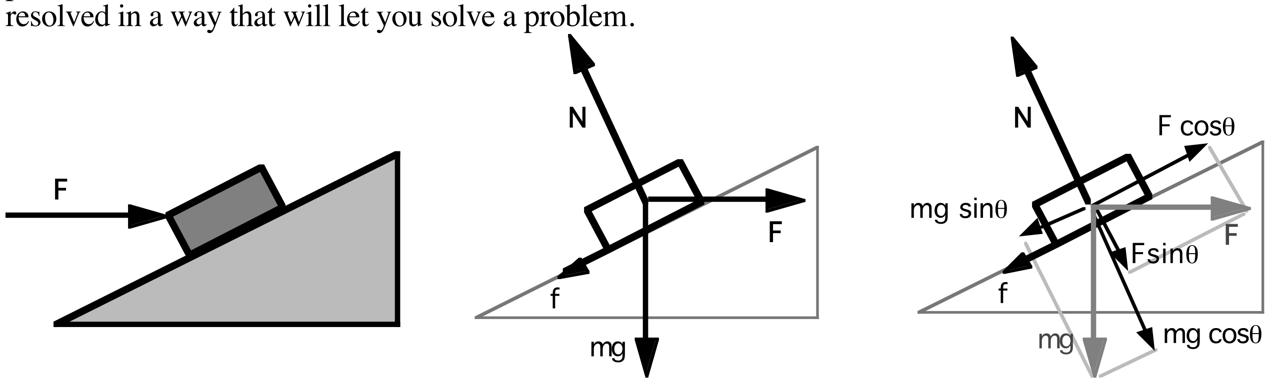
- Often a "three dimensional" projection drawing will help in visualizing a problem and then a 2D figure with vectors attached will show the forces that act on the object being described. I show the velocity or acceleration as detached vectors so that they are not confused with forces.



- Sometimes a series of figures will make the forces much clearer. Use the more elaborate figure set the stage, then simpler drawings with the forces shown can illustrate the point that is being made. In the force diagram figures, show the objects subject to the force in solid lines and de-emphasize the other objects. For example, the third law requires equal forces between hammer and nail:



- Show forces both unresolved and resolved. For example, in the figure below, (a) describes the problem; (b) shows the forces that act <u>on the block</u>; and (c) shows the forces on the block resolved in a way that will let you solve a problem.



SETTING UP NEWTON'S LAW PROBLEMS

"*The formulation of a problem is often more essential than its solution.* "

- *Albert Einstein*

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Solving any problem first requires that it be properly set up - and that requires a careful understanding of the nature of problem, what exactly you are trying to solve for, what forces act on each object whose motion you want to describe, and the cause of each force. A careful set of figures are essential to a correct problem set-up.

In general, since all forces are vectors, Newton's second law must be written in vector form:

$$\vec{\mathbf{F}}_{\mathbf{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

But if the forces and the resulting motion of the object or objects being described are collinear, the problem can quickly be set up as a one-dimension problem and the vector equation can be replaced by its scalar version. But even if the motion must be described in two or three dimensions, Newton's second law can be resolved into its components:

$$F_{net,x} = \sum F_x = ma_x; \quad F_{net,y} = \sum F_y = ma_y; \quad F_{net,z} = \sum F_z = ma_z$$

The choice of coordinates when setting up force problems is an important part of solving problems. In most cases, the motion can be reduced to only two dimensions by simply selecting the coordinates carefully. Then setting the x and y axes simply requires identifying what direction the resulting *acceleration* is in. That is, if one of the axes (say, the *x*-axis) is aligned with the direction of the acceleration of the object, then the net force in the other direction must be zero. Hence the equations of motion can be reduced to just two equations:

$$F_{net,x} = \sum F_x = ma \ ; \quad F_{net,y} = \sum F_y = ma_y = 0$$

Force Diagrams

- Draw all forces that act on each object being described. It is always preferable to draw a separate force diagram (or free-body diagram) for each mass being described – even if they are connected.

- Draw only the forces that act directly on the object you are describing. Do not draw the forces that it exerts on something else or other forces in a system that do not directly act on the mass being described

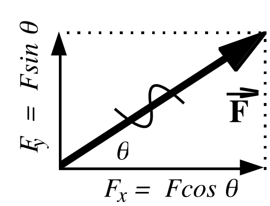
- Draw only forces. Do not attach vectors representing velocities or accelerations to the objects themselves. If you want to show the direction of motion or acceleration adjacent to the object, that's okay, as long as it is clear that is not an additional force to be considered in the problem.

- Label the forces with a notation that you will use in the equations - that way you can check the force equations you eventually write down to be sure you have incorporated everything in the mathematical description.

- Identify (at least mentally) what causes each force you draw. If you cannot decide what is causing some force - that force probably does not exist! [For example, what force is in the direction of motion of a projectile keeping it on its path? ..... I don't know either!]

- Determine the axes onto which you will resolve the forces *for each mass*. Select the axes so that one of the axes is in the direction that mass will accelerate and the other axis is perpendicular to that direction. That will allow you to write the force equations with the net force (or the sum of the components) in one of the directions equal to zero. For example, $\Sigma F_x = Ma_x$ and $\Sigma F_y = 0$ if the acceleration is in the x-direction.

- If you resolve a force into its components on a diagram - be clear that you have done so by showing the vector force and its components with the proper magnitudes and directions and by "eliminating" the original vector in some way. Be sure to draw the force vectors with magnitudes that are appropriate for the forces associated with the problem (eg, if the frictional force is less than the normal force, the diagram should be drawn accordingly). And show the components with appropriate lengths as well, as shown here.



- Be careful about using symbols "loosely" - like using "g" for gravity or "μ" for friction in a force diagram. Let "g" always be the *value* of the freefall acceleration due to gravity - not just a symbol for the concept of gravity and certainly not a force. That is $F_g = mg$ is proper, but just g is not. Similarly, "μ" is just the coefficient that relates the frictional force between two surfaces to the normal force N that presses the two surfaces together. So $F_{friction}$ or F_f or just $f = \mu N$ is proper.

- Write a force equation for each mass. After constructing a carefully drawn force diagram for each object whose motion you want to describe - write the equation for that object that relates the forces that act directly on it and the resulting acceleration. If you need to deal with motion in two dimensions then write the equations in component form: $\Sigma F_x = Ma_x$ and $\Sigma F_y = Ma_y$

Only consider solving for some quantity (tension in the string, acceleration, or whatever) after you are sure you have included all the forces in both the diagrams and the equations. Be sure your force equations are consistent with the force diagrams. [For example, if an object is being accelerated upward by the tension in a string, then $T - mg = ma$ describes correctly that the object would have a positive value for the acceleration a as long as $T>mg$.]

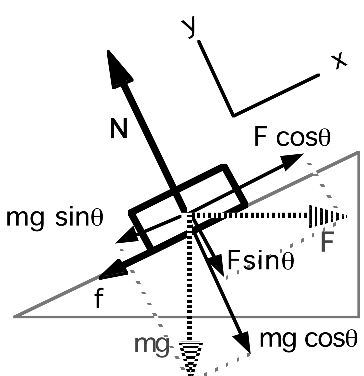
Setting up a problem incorrectly can never lead to a correct solution. You might stumble on a correct result for a particular special case problem - but your approach will not help you solve other problems.

Do not let what you think might happen supercede the systematic use of principles in solving a problem.

The important point here is that once all of the forces on some object are identified - both in magnitude and direction, the net force is just the vector addition of those forces and the acceleration that results can be used to then solve for the motion of the object. With a correctly drawn force diagram, writing the correct equations to describe the force-acceleration relationship should simply be a matter of "reading" your diagram.

Example:

Consider the problem of the block being pushed up an incline plane by a horizontal force **F** described earlier. Notice that there is a choice to be made for the direction of the x- and y-axes in this problem. That is, we could choose x and y to mean the horizontal and vertical directions, or we could choose x and y to mean parallel and perpendicular to the plane on which the block movement occurs. As long as the block stays in contact with the incline, there is no acceleration perpendicular to the incline. That should dictate the choice of which set of axes to use in describing the problem.



First, the forces are identified - ie, the gravitational force mg, the horizontal applied force **F**, the normal force **N**, and the frictional force **f** (which we have assumed to be down the incline - a choice made by the problem statement which says it is moving up the incline). Then the axes are chosen so that one of the axes corresponds to the direction of any acceleration that might occur (ie, along the incline) so that the acceleration is zero along the other axis (ie, perpendicular to the incline). Finally, the identified forces are resolved into components parallel to the incline and perpendicular to the incline. The only task that remains is to write the force equations for each of the component directions. The force equations are nothing more than Newton's second law for each component.

The force equations:

x-direction: $\sum F_x = F\cos\theta - mg\sin\theta - f = ma_x$ (parallel to plane)

y-direction: $\sum F_y = N - mg\cos\theta - F\sin\theta = ma_y = 0$ (perpendicular to plane)

Solving the y-equation for the normal force $N = mg\cos\theta + F\sin\theta$ and then noting that the frictional force *f* is related to the normal force by $f = \mu_k N$ allows solving the x-equation for the relationship between the acceleration of the block and the various forces that act on the block.

$$F\cos\theta - mg\sin\theta - \mu_k(mg\cos\theta + F\sin\theta) = ma_x$$

Notice that as complicated as this result seems to be, once the force diagram was drawn correctly, and the components were identified, the rest of the problem just involves "reading" the force diagram! Also notice that if the resulting calculated acceleration is positive, it means the acceleration is up the incline (dictated by the choice of the direction of the positive x-direction made when setting up the equations). But the method of solution also allows for the acceleration to be negative, ie, down the incline which would mean a slowing of the block, even if it is sliding *up* the incline.

It is important to set up many problems until it is clear how to carefully construct force diagrams that will help you then write the force equations. The key is to establish a coordinate system for every moving object – with the direction of one axis along the direction for which no acceleration (or, in some cases, any movement at all) occurs. The sum of all the components of forces acting along that direction would then necessarily be zero. The sum of forces along the other axis would then equal the object's mass times its acceleration. If that process is followed, it should be easy to write the force equations by simply "reading" the force diagram.

CIRCULAR MOTION PROBLEMS

When objects move in curved paths, as discussed in Ch. 2 on kinematics, there is always a component of the acceleration perpendicular to the direction of motion – *ie,* there is always a *centripetal* acceleration, toward the center of curvature of the path being taken. That acceleration must be a result of a component of the net force in the same direction. So in all problems which involve curved paths, there must be a net force with a component toward the center of curvature.

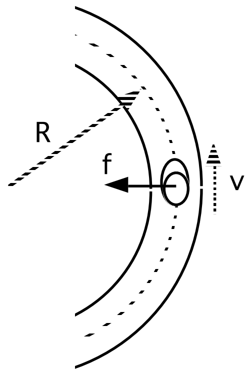
It is useful to examine several examples of circular motion problems – all of which reinforce the essential idea of Newton's second law, that the acceleration of an object is in the direction of the net force. If that net force is perpendicular to the motion of an object, then the resulting motion will be in a curved path.

Consider three problems: (1) a car negotiating a circular path through a turn; (2) a "conical pendulum" – a ball on a string which is being swung in a horizontal circle; (3) a planet orbiting the sun. In all three cases, the net force is perpendicular to the velocity of the object if is it is traveling at constant speed. Since that speed is continuously changing direction, the corresponding acceleration is toward the center of the circular path.

Car in a circular path

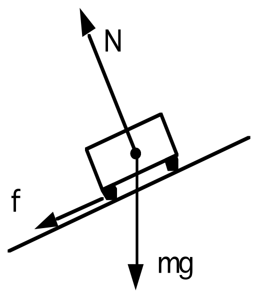
First assume the roadway is unbanked – so the only horizontal force acting on the car is the friction of the road on the tires. Since the frictional force is the only force in the direction of the acceleration, it is the net force and is numerically equal to the mass of the car times its acceleration toward the center – *ie,* the *centripetal acceleration*.

$$F_{net} = f = \mu mg = ma_c$$



where the centripetal acceleration is $a_c = \frac{v^2}{R}$

If the turn is banked, the problem is more complicated because you need to keep track of the vertical and horizontal components of both the normal force and the frictional force. The *net* horizontal force in that case is the sum of the two horizontal forces. That is, both the banking and the frictional force contribute an inward force on the car which helps it negotiate the turn. (In the figure, the car is being viewed from behind – and the gravitational force, the normal force, and the friction of the road on the tires is shown. Since the car is traveling in a *horizontal* circle, the acceleration must be horizontal toward the center of the circular path. The horizontal components of the frictional and normal forces thus add to equal the net force, which in turn equals the car's mass times the centripetal acceleration. That is



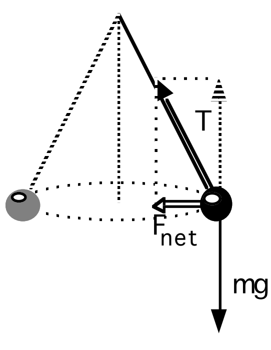
$$F_{net} = N_x + f_x = ma_c = m\frac{v^2}{R}$$

The horizontal and vertical components of the forces can be determined by a resolving the forces and recognizing that the vertical components must add to zero – since the car does not accelerate perpendicularly to the track surface. That is,

$$N\cos\theta = mg + f\sin\theta \qquad \text{and} \qquad N\sin\theta + f\cos\theta = m\frac{v^2}{R}$$

where the force due to friction, if the car is at its limit of adhesion is $f=\mu_s N$.

Conical Pendulum

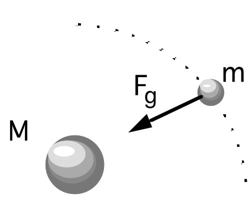


This problem is quite similar to the banked turn problem in the absence of friction, That is, the only two forces acting are gravity and the string tension. So, again, since the pendulum bob is moving in a horizontal circle, the vertical component of the tension must "balance" the gravitational force and the horizontal component of the tension is the only unbalanced force – hence is the net force. And the consequence of the net force is the centripetal acceleration associated with its circular motion. That is,

$$F_{net} = T_x = T\cos\theta = m\frac{v^2}{R}$$

where R is the radius of the circle and $T\sin\theta = mg$.

Gravitational Orbits



It is in this problem that we see the brilliant synthesis that was done by Isaac Newton in the seventeenth century. That is, he combined what he knew about circular motion, that the acceleration had to be v^2/R, the idea that all accelerations are the result of the net force that acts on an object, and the notion that the only force on an planet or moon as it orbits is the gravitational pull of its "host". That is, if the only force acting on the moon of mass m is due to the Earth of mass M a distance R away is given by the universal law of gravity and the orbiting moon is in a circular orbit, the acceleration must be just the centripetal acceleration associated with the moon's orbit. So

$$F_g = G\frac{mM}{R^2} = m\frac{v^2}{R}$$

Since the moon's mass cancels in this equation and both the distance to the moon and the speed of the moon in its orbit can be known (*ie*, $v=2\pi R/T$, where T is the period of its orbit), the mass of the Earth can be determined from observing the moon's orbit! Moreover, this idea applies to every planet orbiting the sun (hence the sun's mass can be determined), and to every other star-planet or double star systems that can be observed in the solar system! It is the way we know the masses of the sun and the planets and of some stars.

In the early 1600s, Johannes Kepler had deduced from observations that the planets orbited the sun in such a way that the square of their orbit periods was proportional to the cube of their orbit radii (technically, the semi-major axis of their elliptical orbits). Combining the information above lets this third law of Kepler to be mathematically derived. That is,

$$G\frac{M}{R^2} = \frac{v^2}{R} = \frac{\left(2\pi R/T\right)^2}{R} \quad \text{which leads to} \quad T^2 = \frac{4\pi^2}{GM}R^3$$

which is just the expression for Kepler's Third Law of planetary orbits.

What is common to all of these problems, of course, is that the net force equals the mass of the object moving in a circle times the centripetal acceleration. And the centripetal acceleration as we saw in the section on circular motion in kinematics only depends on the object's tangential speed and the radius of the circular path.

NEWTON'S LAWS – QUESTIONS AND PROBLEMS

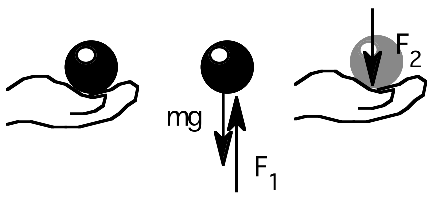
1. If two objects are shot horizontally with different initial speeds: [SELECT AND EXPLAIN]

   The faster one lands first due to its greater speed.
   The slower one lands first due to the shorter distance traveled.
   They are in the air for the same amount of time.

2. If an object moves in a uniform circle at a constant speed subject to several forces (gravity, normal force, friction, etc.), then the net force: [SELECT AND EXPLAIN]

   i. Must have a component toward the center of the circle.
   ii. Must be a vector toward the center of the circle.
   iii. Is a vector whose direction can only be determined when the magnitudes and directions of all the forces acting are considered.

3. Suppose you move an object of mass m vertically. For each of the following cases, compare:

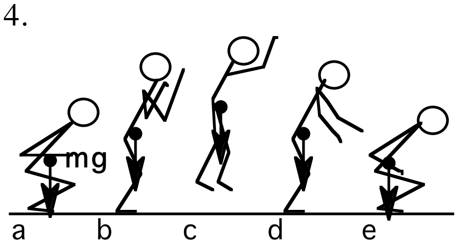


a. The force you exert on the object (**F_1**) and the gravitational force acting on the object (**F_g=mg**); and

b. The force **F_1** and the force the object exerts on you (**F_2**) Explain your thinking carefully.

   i. You lower the object to the ground at constant speed.
   ii. You lift the object with increasing speed.
   iii. You lift the object with decreasing speed.
   iv. You catch the object as it is dropped into your hand, bringing it to a stop.

4.



Consider that you jump vertically upward. Compare the normal force the floor exerts on you **N**, the gravitational force on you **mg**, and the force you exert on the floor **F** for each of the four cases mentioned:
[Make your comparisons using “>”, “<”, and “=” signs.]

Complete the figures by adding the normal force vectors **N** to the sketches (in the proper proportion).

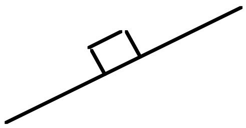
a. Before you start your jump (fig. a):	(a) **N** **mg**	(b) **F** **mg**	(c) **N** **F**
b. While you are jumping (fig. b):	(a) **N** **mg**	(b) **F** **mg**	(c) **N** **F**
c. Just after your feet hit the floor while landing (fig. d):	(a) **N** **mg**	(b) **F** **mg**	(c) **N** **F**
d. After you come to rest (fig. e):	(a) **N** **mg**	(b) **F** **mg**	(c) **N** **F**

5. a. The following are incorrect statements of Newton's 2nd and 3rd laws. Briefly explain why the statements are incorrect and then rewrite the sentence so that is a correct statement of either Newton's 2nd or 3rd law.

   i. Any force acting on an object will result in an acceleration.

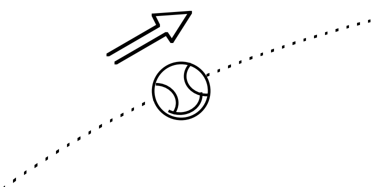
   ii. Any force acting on an object will be accompanied by an equal and opposite force on the same object.

6. A horizontal force F pushes a block of mass M up an incline which makes the angle θ with the horizontal. The coefficient of kinetic friction between the block and incline plane is given by μ. Carefully construct the force diagram.

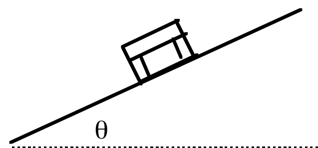


Set up the equations for the forces parallel and perpendicular to the incline that would let you solve for the acceleration of the block. [Your expressions will contain the quantities F, N, M, g, μ, and θ.]

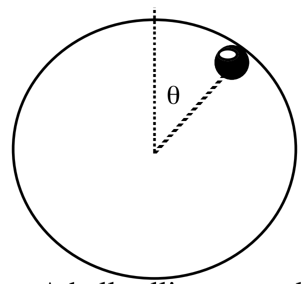
7. Carefully draw the all forces acting for each of the following cases and identify what each of the forces you identify are due to. Write the appropriate equations that relate the forces to the motion.



A baseball in mid-flight

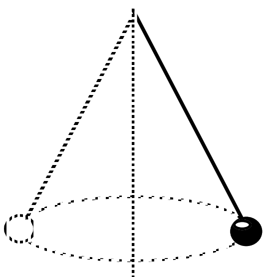


A piano sliding down an incline with friction.

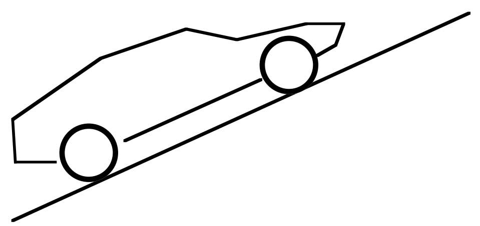


A ball rolling around inside a circular track

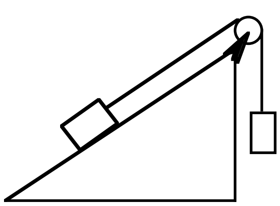
8. Carefully draw the forces that act on each mass and identify their causes. Relate the forces to the motion – that is, write the equations relating the forces to the resulting accelerations along the apprpriate axes).



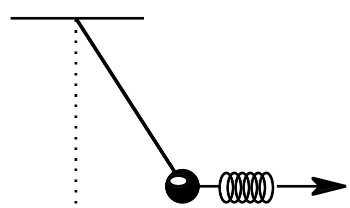
A ball on a string moving as shown



A rear-wheel drive car accelerating up an incline

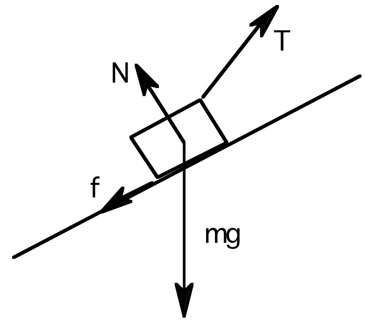


Two masses over a pulley as shown



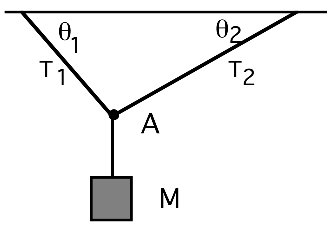
A spring holds a ball on a string in equilibrium

9. The figure shows the forces that are acting on a block on an incline. Assume that the forces are drawn with their correct magnitudes and directions.



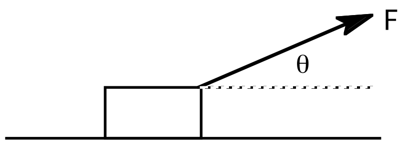
Describe the motion of the block. That is, state whether it is moving up or down the incline (or is at rest) and whether it is traveling at constant speed or is increasing or decreasing its speed. Explain your reasoning carefully.

10. For the suspended mass, draw the forces that act (a) on the mass; and (b) at point A.



W rite the force equations that would let you solve for T_1 and T_2 if you knew the angles and the mass.

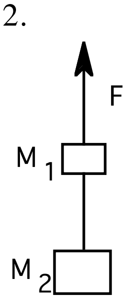
11. A block of mass M is being pulled across a surface by a cord at an angle q with the horizontal as shown. The coefficient of friction is μ. Assume you know F, M, g, μ, and θ.



Carefully draw the force diagram and write the equations which would allow you to solve for the acceleration and the normal force.

Find an expression for the angle for which the force F to slide the block at constant speed is minimized.

12.

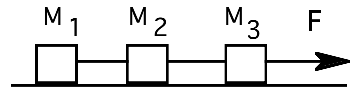


Consider two masses M_1 and M_2 which are connected by a string, as shown. Suppose a force F accelerates the "system" vertically. Assuming the force F is known, write an expression that would allow you to determine the acceleration of the blocks.

Consider each mass separately. On the figures, show the forces that act on each mass. Label each of the forces and write a separate equation for each mass that would give its acceleration.

Suppose the rope had mass m, how would that change the equations and the solution?

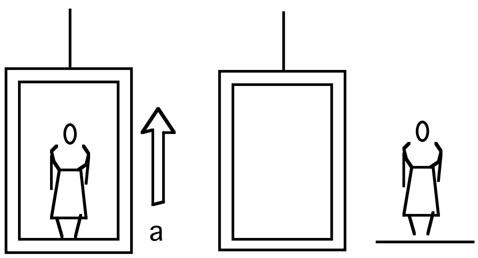
13. Consider three blocks of masses M_1, M_2, and M_3 connected by ropes as shown. The system is being accelerated to the right on a frictionless horizontal surface by the force **F**.



Write expressions for the acceleration of the system and tensions in the all the ropes. Explain carefully.

Now modify the equations to account for friction if the coefficient of kinetic friction between blocks and surface is μ_k

14.

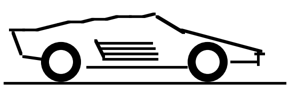


Consider that Ethel, mass m, is on an elevator of mass M which is accelerating upward.

Carefully draw all the forces that act on the elevator, and all the forces that act on Ethel. Write the corresponding equations which would let you solve for the tension in the cable (T) and the normal force (N) between the floor and Ethel if the masses and acceleration are known.

15. Your Ferrari can accelerate from zero to 60 mph (30 m/s) in five seconds. Let M=1200 kg.

a. Determine the average force required to accelerate the car at that rate.



On the figure, show all of the forces acting on the car during acceleration.

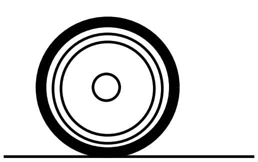
Determine the minimum coefficient of friction between the rear tires and pavement required to result in the calculated acceleration assuming 2/3 of the car's weight is on the rear (driving) wheels.

c. Assuming the coefficient of static friction between the tires and pavement is μ_s=1.0, determine the minimum stopping distance from 60 mph (or 30 m/s).

Explain carefully why the stopping distance is shorter when the tires are not allowed to skid on the pavement.

d. Assuming the same coefficient of friction, determine the maximum speed with which the Ferrari can negotiate a turn with a radius of 40 m, assuming the curve is not banked.

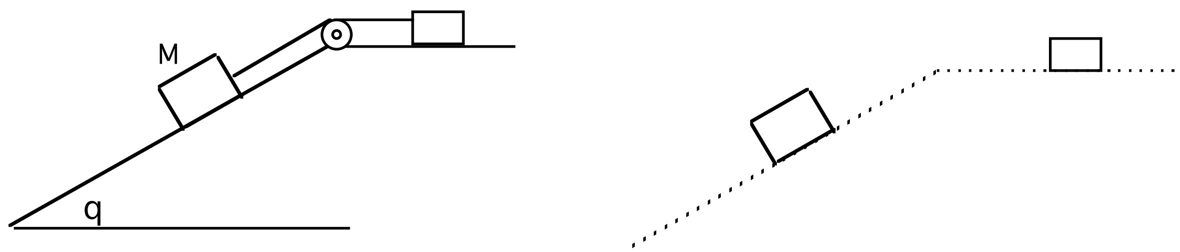
16. The wheel of a car rolls along the pavement at constant speed. Show the forces that act on the wheel.



Suppose the wheel is one of the *driving* wheels (eg, a rear wheel on a rear wheel drive car) and the car accelerates. Show the forces on the wheel.

Now suppose the driver hits the brakes, and the car slides to a stop. Show the forces on the wheel.

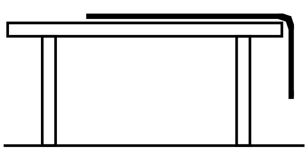
17. A block of mass M is on an incline which makes an angle q with the horizontal. It is connected to a second block of mass m on the horizontal surface as shown. The coefficient of kinetic friction between the blocks and surfaces is given by μ. (a) Carefully draw and label all the forces that act on each block. Include the appropriate force components parallel and perpendicular to the incline.



(a) (b)

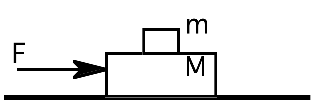
Set up the equations that would allow you to determine the acceleration of the blocks and the tension in the string. [Your expressions should only contain the unknowns T and a and the quantities m, M, μ, q and, of course, g.]

18.



Consider a rope of length L that lies on a table with an amount y hanging over the side. If the coefficient o f static friction between the rope and the table is given by μ, determine what fraction of the length of the rope must hang over the side for it to begin to slip.

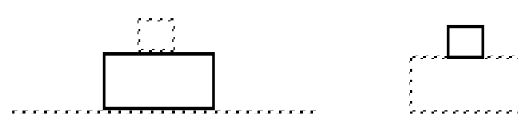
19. Consider a table with two blocks on it as shown. Let the coefficient of friction between M and the table be μ=0.6 and that between the small block m and the large block be μ=0.4. (Assume the static and kinetic coefficients are the same in both cases.)



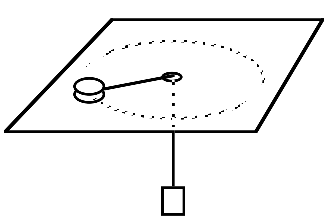
Consider that a force F is applied as shown.

a. Obtain an expression for the maximum force that can be applied to M without m sliding on the larger block M. [Hint: Determine the maximum acceleration that m can have given the only force that acts on it.]

b. If F is large enough, M can accelerate out from under m - ie, m slides on M. Draw the forces that act on each block and write the equations that would allow you to determine the acceleration of each block and the contact force that acts between the blocks.

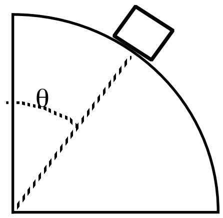


20. Consider a horizontal table with a hole in it. A block moves in a circle on the table. A string attached to the block supports another block below the table (through a hole) as shown. There is friction between the block and table.



Draw all the forces that act on each block and write the equations which relate the masses, forces, velocity, and accelerations, etc.

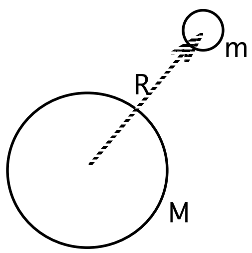
21. An ice cube slides off a hemispherical surface (frictionless).



Draw the force diagram when it is still in contact, traveling at speed v and is at the angle q with respect to the vertical.

Write the two force equations which relate the normal and gravitational forces to the speed, radius, and angle.

22. Consider that the gravitational force between any two objects of masses m and M is given by



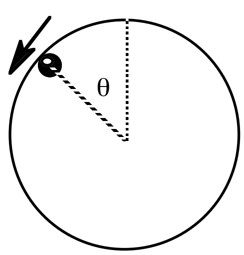
$$F_g = G\frac{mM}{R^2}$$

where G is a universal constant ($6.7\text{x}10^{-11}$ N-m^2/kg^2) and R is the separation between the centers of the masses.

a. Knowing the value of g, look up the radius of the earth-moon orbit and determine the mass of the earth.

b. The diameter of earth's moon is one-fourth that of earth itself. Its mass is about 1% of earth's mass. Determine the acceleration due to gravity at the surface of the moon.

23.

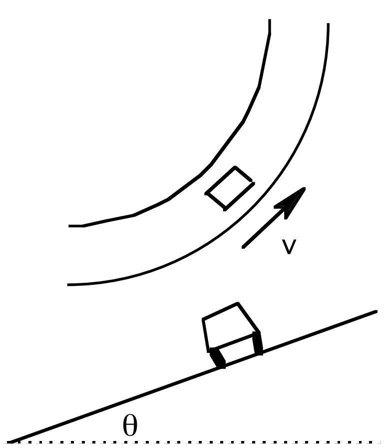


A ball moves inside a vertical circular track. Draw all the forces that act on the ball. Relate the forces acting on the ball to the speed of the ball when it is at the position shown. (You may ignore friction.)

Determine the minimum speed the ball can have and still be in contact with the track at the top.

When the ball goes through the minimum position, compare the gravitational force on the ball to the normal force of the track on the ball. Explain carefully.

24. Suppose a car of mass M travels around a banked constant radius turn at a constant speed v.



a. Obtain an expression for the maximum speed with which the car could negotiate the turn without slipping assuming a static coefficient of friction μ between the tires and pavement and a banking angle q with respect to the horizontal. Carefully draw the force diagrams (in both views) and set up the problem carefully. Explain your work.

b. Examine your expression for maximum speed through the turn for the two cases: (a) $q = 0$ to find the minimum coefficient of friction on a flat turn, and (b) μ=0 to find the one speed that you could make it through without friction.

c. Obtain an expression for the maximum speed with which the car can negotiate the turn of radius R if the banking angle is and the coefficient of static friction is μ.